GROUNDWATER FLOW SYSTEM MODELS AND INTERPRETATION OF VARIABLE INTERACTIONS AT GUASCA MUNICIPALITY IN COLOMBIA

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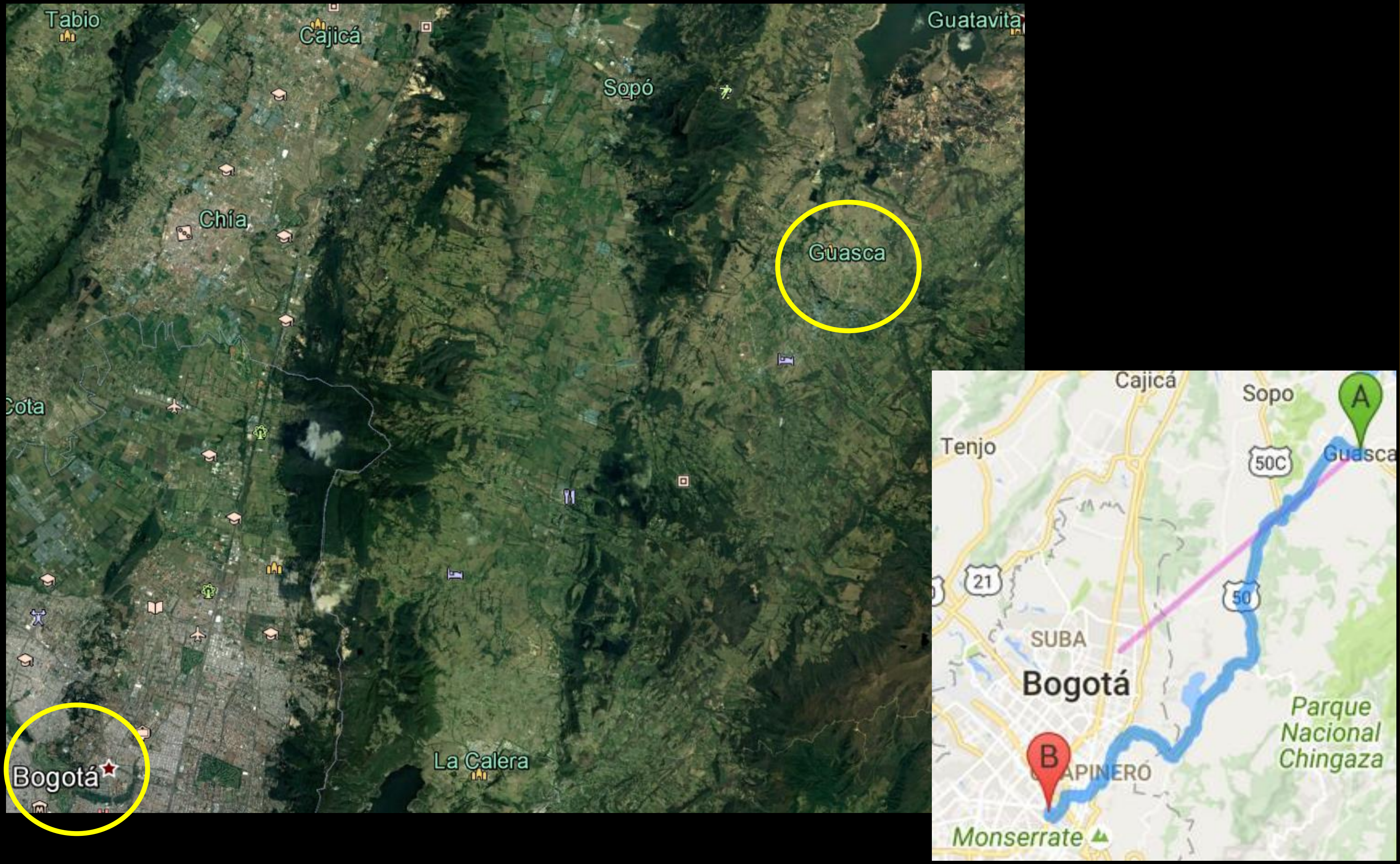
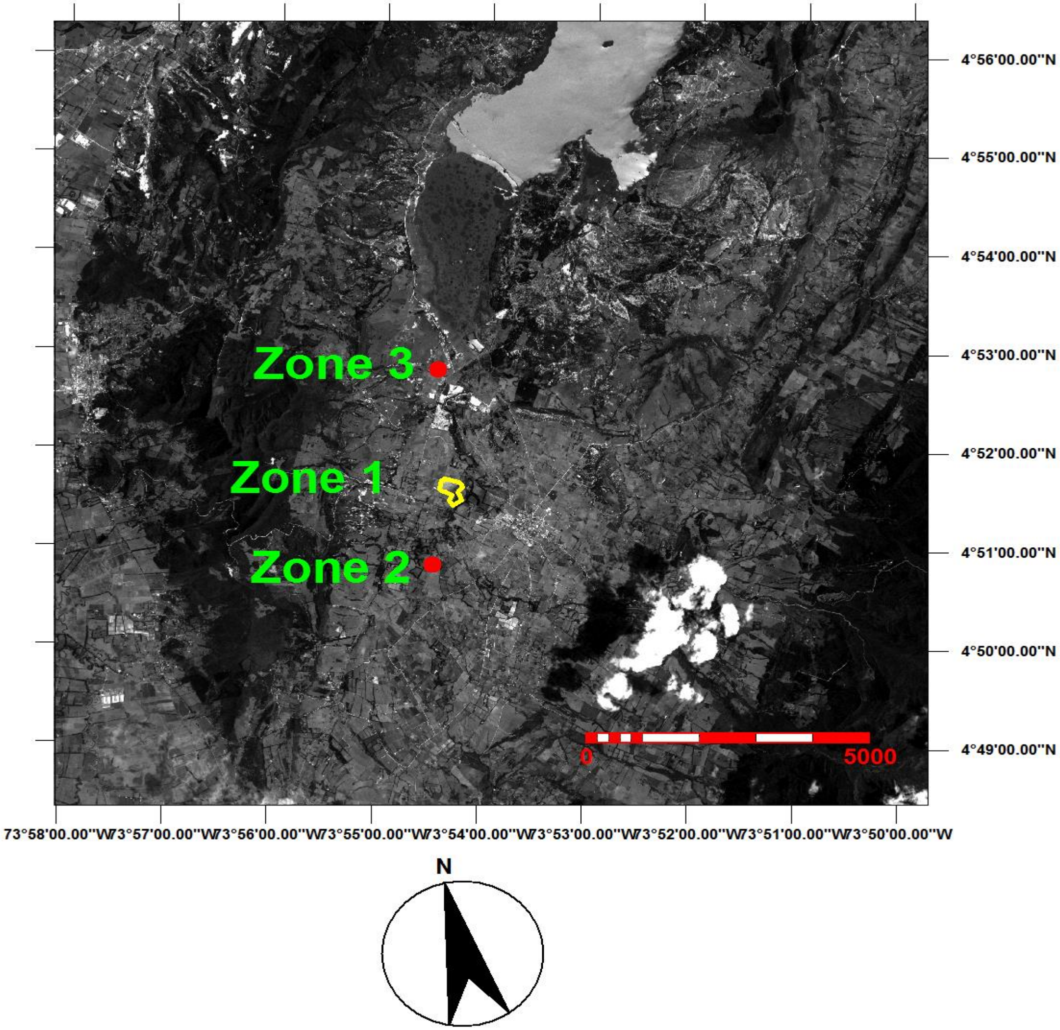
OUTLINE

- Introduction
- Procedure
- Results
- Conclusions

INTRODUCTION

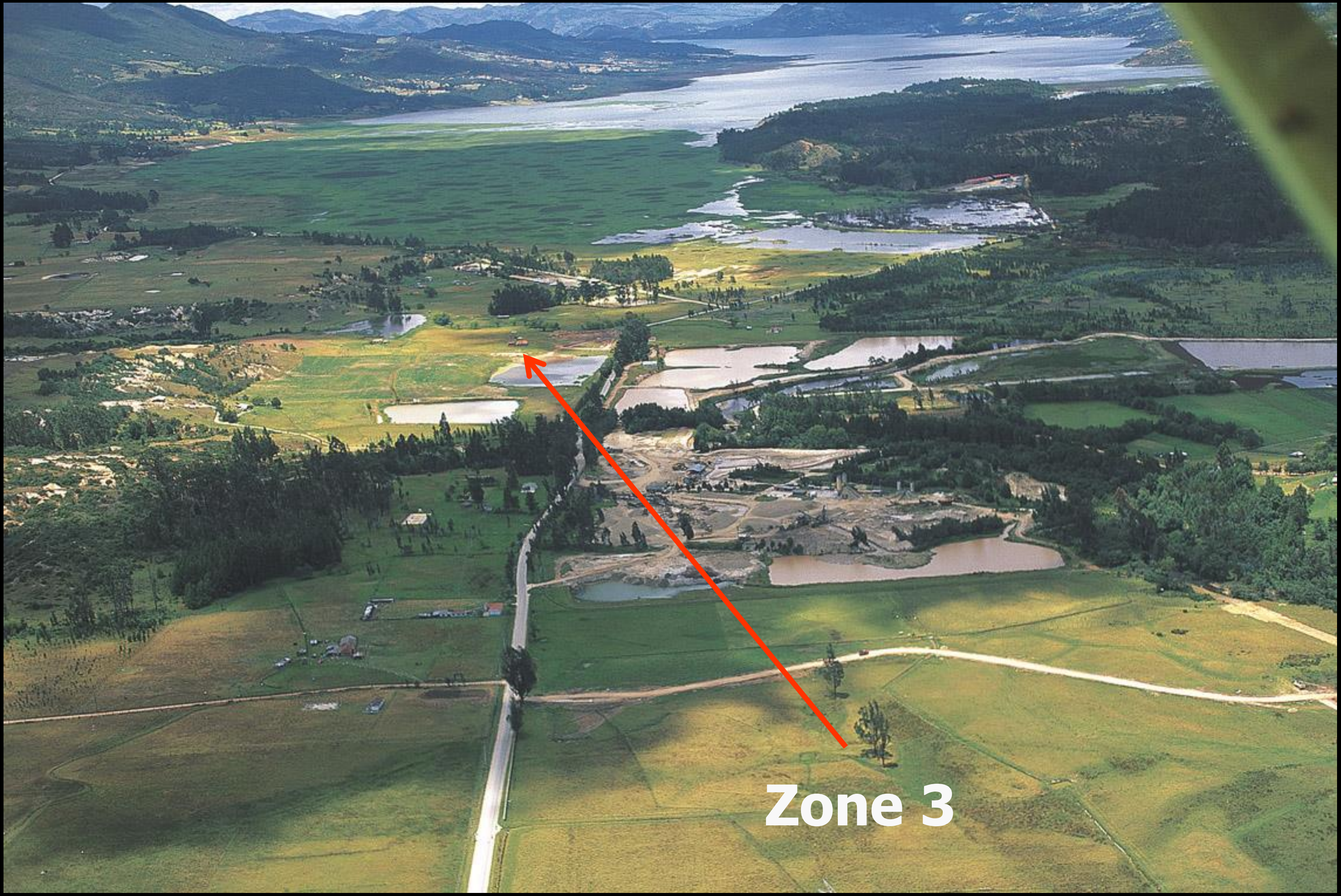


LOCATION OF THE STUDIED ZONES AT GUASCA MUNICIPALITY IN COLOMBIA

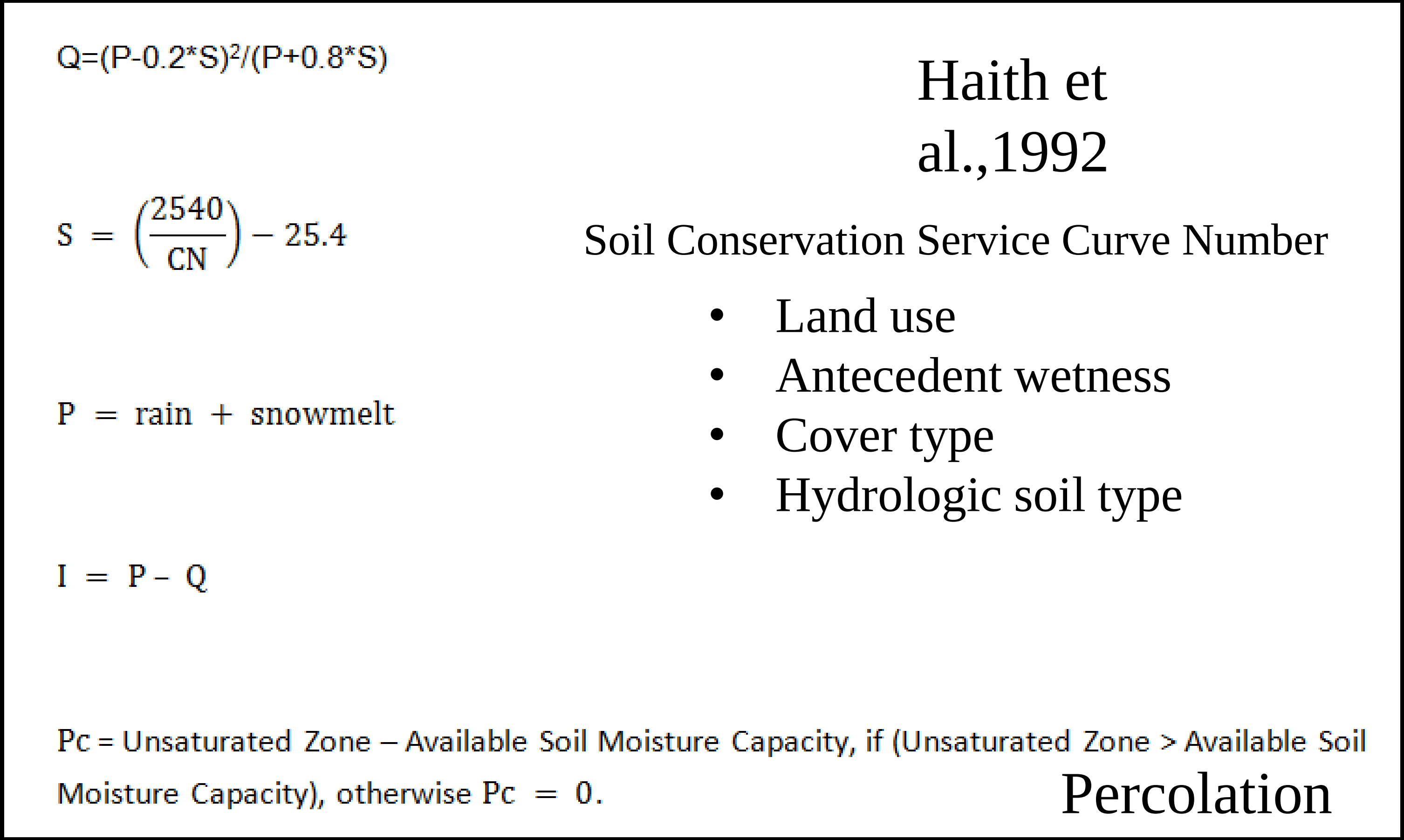


Zone 1

Zone 2


Zone 3

The Generalized Watershed Loading Functions (GWLF) model



$Q=(P-0.2*S)^2/(P+0.8*S)$

$$S = \left(\frac{2540}{CN}\right) - 25.4$$

$P = rain + snowmelt$

$I = P - Q$

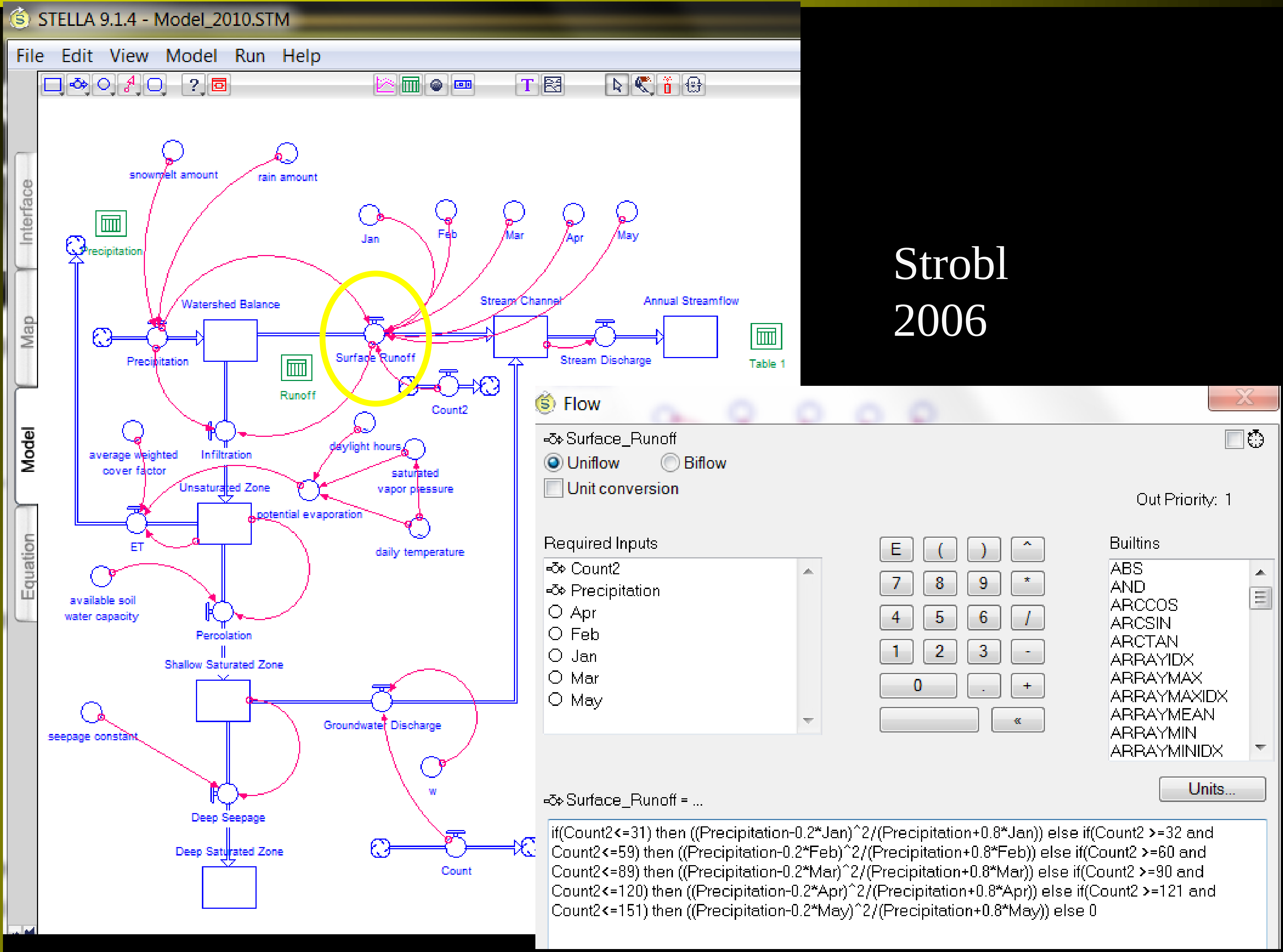
Haith et al.,1992

Soil Conservation Service Curve Number

- Land use
- Antecedent wetness
- Cover type
- Hydrologic soil type

Pc = Unsaturated Zone – Available Soil Moisture Capacity, if (Unsaturated Zone > Available Soil Moisture Capacity), otherwise Pc = 0.

Percolation



Surface_Runoff

● Uniflow ○ Biflow

☐ Unit conversion

Out Priority: 1

Required Inputs

- Count2
- Precipitation
- Apr
- Feb
- Jan
- Mar
- May

Surface_Runoff = ...

```
if(Count2<=31) then ((Precipitation-0.2*Jan)^2/(Precipitation+0.8*Jan)) else if(Count2 >=32 and
Count2<=59) then ((Precipitation-0.2*Feb)^2/(Precipitation+0.8*Feb)) else if(Count2 >=60 and
Count2<=89) then ((Precipitation-0.2*Mar)^2/(Precipitation+0.8*Mar)) else if(Count2 >=90 and
Count2<=120) then ((Precipitation-0.2*Apr)^2/(Precipitation+0.8*Apr)) else if(Count2 >=121 and
Count2<=151) then ((Precipitation-0.2*May)^2/(Precipitation+0.8*May)) else 0
```

Strobl
2006

The Generalized Watershed Loading Functions (GWLF) model

$$\text{Unsaturated Zone} = \text{Infiltration} - \text{Evapotranspiration} = I - ET$$

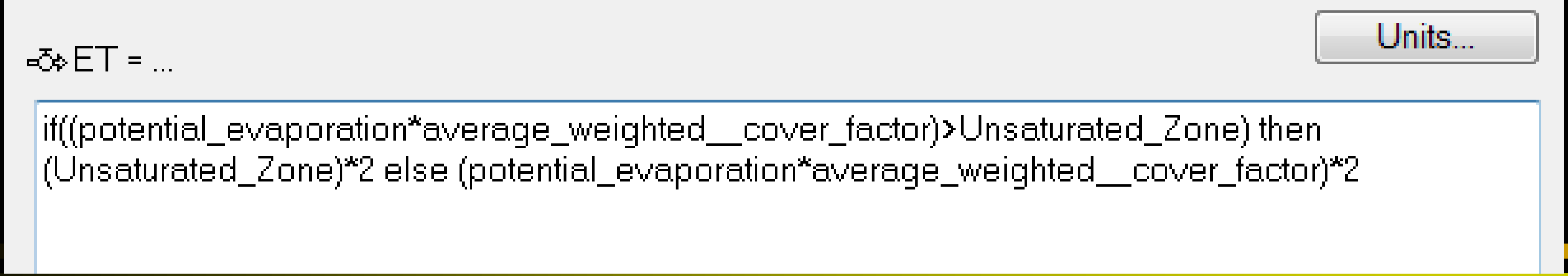
$$ET = PE \times CV, \text{ if } [(PE \times CV) > \text{Unsaturated Zone}], \text{ else } ET = \text{Unsaturated Zone}.$$

$$PE = \frac{0.021\, H^2\, e}{(T + 273)}$$

H= number of daylight hours

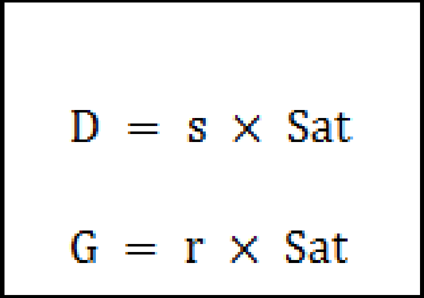
$$e = 33.8639\, [(0.00738\, T + 0.8072)^8 - 0.000019\, (1.8\, T + 48) + 0.001316]$$

Saturated water vapor pressure



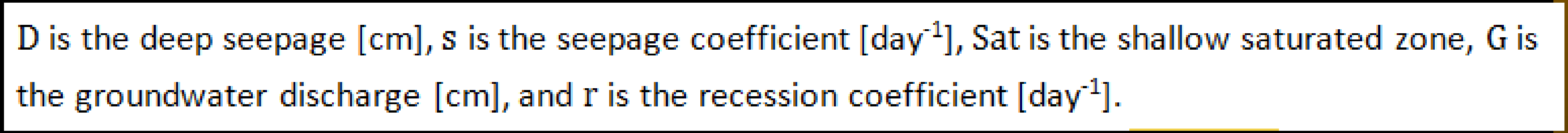
```
if((potential_evaporation*average_weighted__cover_factor)>Unsaturated_Zone) then
(Unsaturated_Zone)*2 else (potential_evaporation*average_weighted__cover_factor)*2
```

The Generalized Watershed Loading Functions (GWLF) model

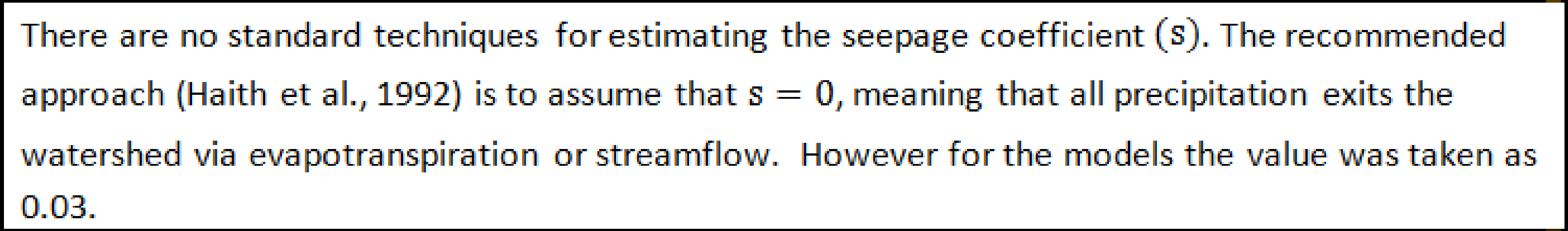


$$D = s \times Sat$$

$$G = r \times Sat$$



D is the deep seepage [cm], s is the seepage coefficient [day^{-1}], Sat is the shallow saturated zone, G is the groundwater discharge [cm], and r is the recession coefficient [day^{-1}].



There are no standard techniques for estimating the seepage coefficient (s). The recommended approach (Haith et al., 1992) is to assume that $s = 0$, meaning that all precipitation exits the watershed via evapotranspiration or streamflow. However for the models the value was taken as 0.03.

During periods of streamflow recession, it is assumed that runoff is negligible. As a result, streamflow $F(t)$ [cm] = $G(t)$, and the recession constant can be estimated from two streamflows $F(t_1)$ and $F(t_2)$, measured on days t_1 and t_2 $(t_2 > t_1)$. The ratio of $F(t_1)$ to $F(t_2)$ is:

$$\frac{F(t_1)}{F(t_2)} = \frac{r\,S(0)e^{-rt_1}}{r\,S(0)e^{-rt_2}} = e^{r(t_2-t_1)}$$

$$r = \frac{\ln\left[\frac{F(t_1)}{F(t_2)}\right]}{t_2 - t_1}$$

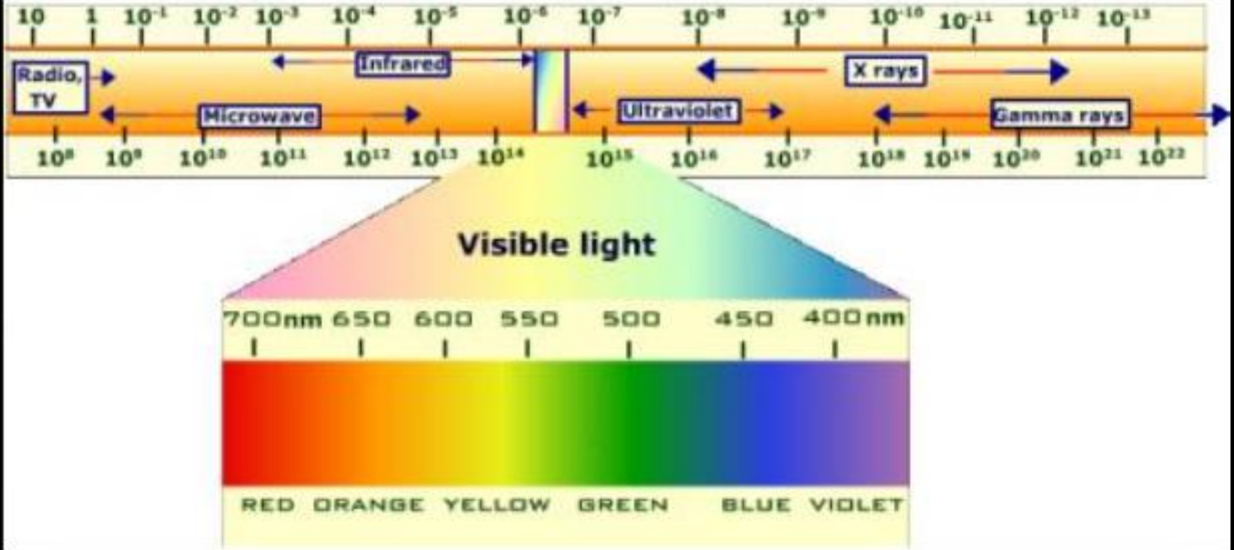
The recession constant was based upon two measured streamflows: one on February 18 2007 with a value of 0.68 $m3s^{-1}$ and February 27 2007 with a correponding value of 0.53 $m3s^{-1}$ as follows:

$$r = \frac{\ln\left[\frac{0.68}{0.53}\right]}{27 - 18} = 0.028$$

The average value of 0.028 were considered in the calibration of the STELLA model. However, that value changed latter per yearly basis, as a result of fitting process. For the year 2008, the value was taken as 0.03129, for the year 2009, the corresponding value was 0.01199 and for the year 2010, the value was 0.03834.

The NDVI responds to changes in the quantity of green biomass, chlorophyll content, and vegetation stress when there is a lack of water (Liang 2004). The NDVI of vegetation ranges from -1 to 1. The NDVI of water varies from -1 to 0 (Zhou et all 2007). NDVI < 0.2 is considered bare soil. NDVI > 0.2 and < 0.5 is a mixture of bare soil and vegetation. NDVI > 0.5 is vegetation only (Julien et all 2006). The greater the NDVI, the greater the phyto-mass and the photosynthetic activity within a specific zone (Timmermans 1995).

$$\mathrm{NDVI} = \frac{R_{nir} - R_{red}}{R_{nir} + R_{red}}$$



PROCEDURE

28 CM 5

Latitude: 04^{o} 53’ 48.817 87” N

Longitude: 73^{o} 52’ 35.766 40” W

Height: 2601.184 m

GAUSS-KRÜGER

North: 1033254.581 m

East: 1022283.725 m

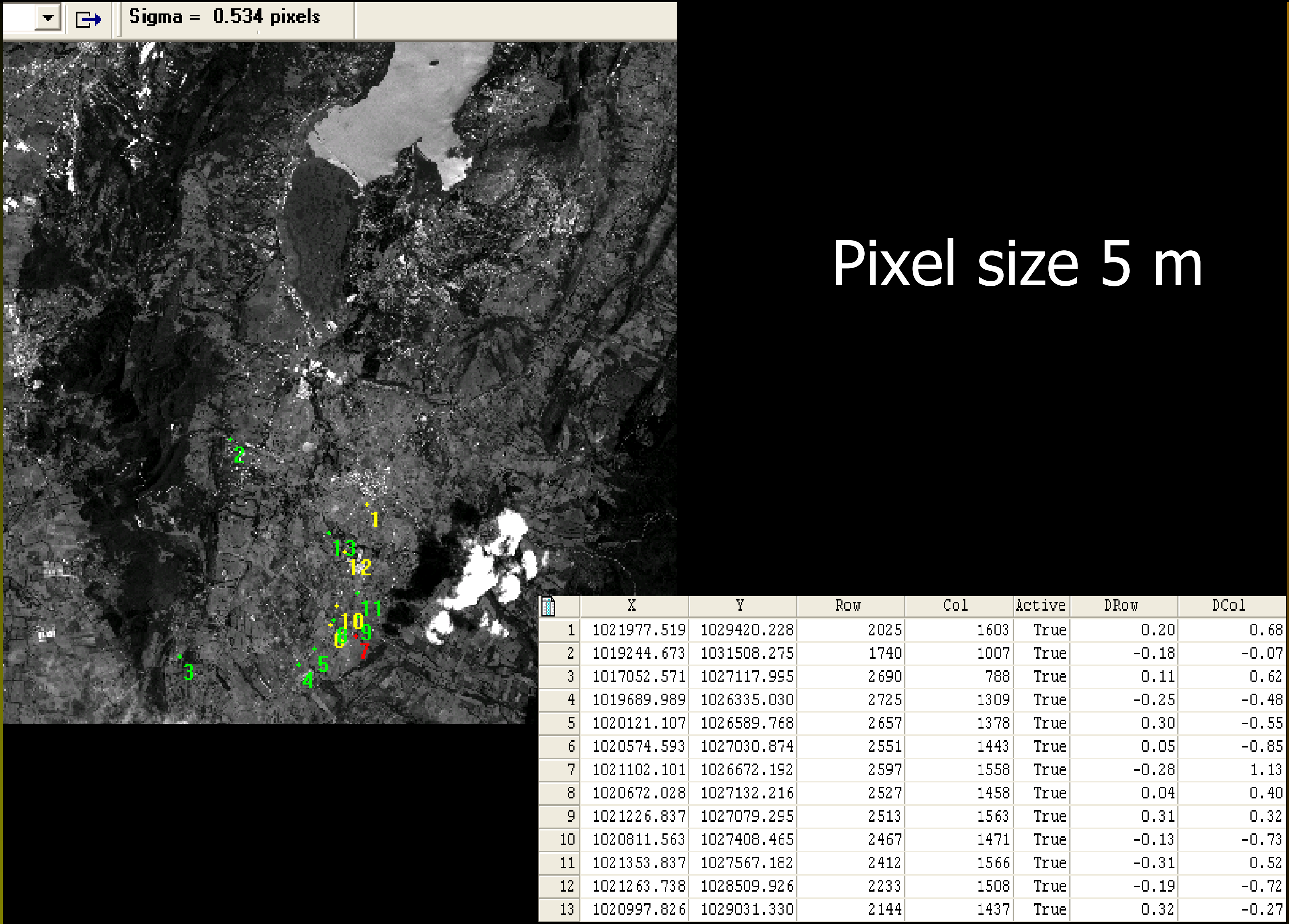
Doble frequency Hipper GGD from TOPCON

NP28 CM-5

GPS-1

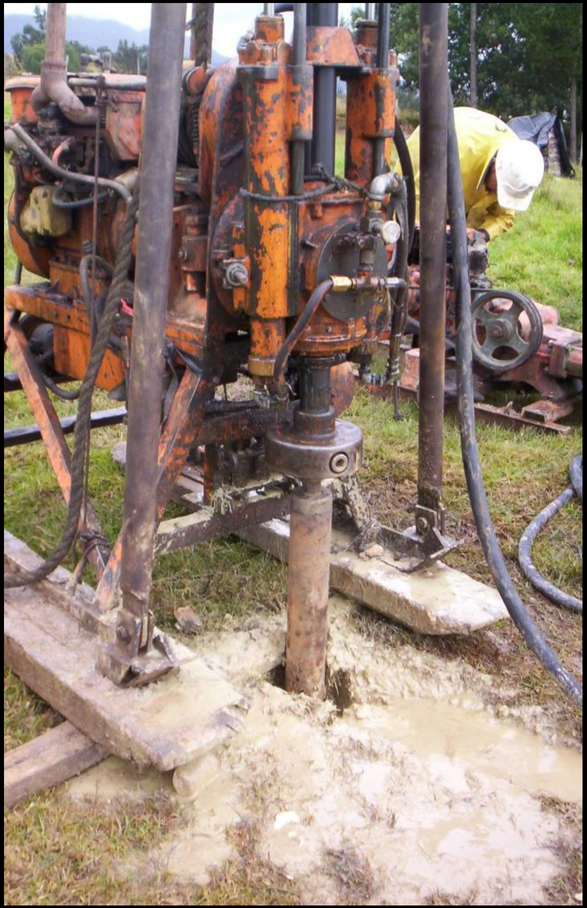
GPS-2

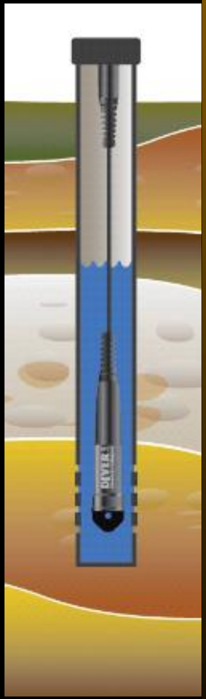
Pixel size 5 m

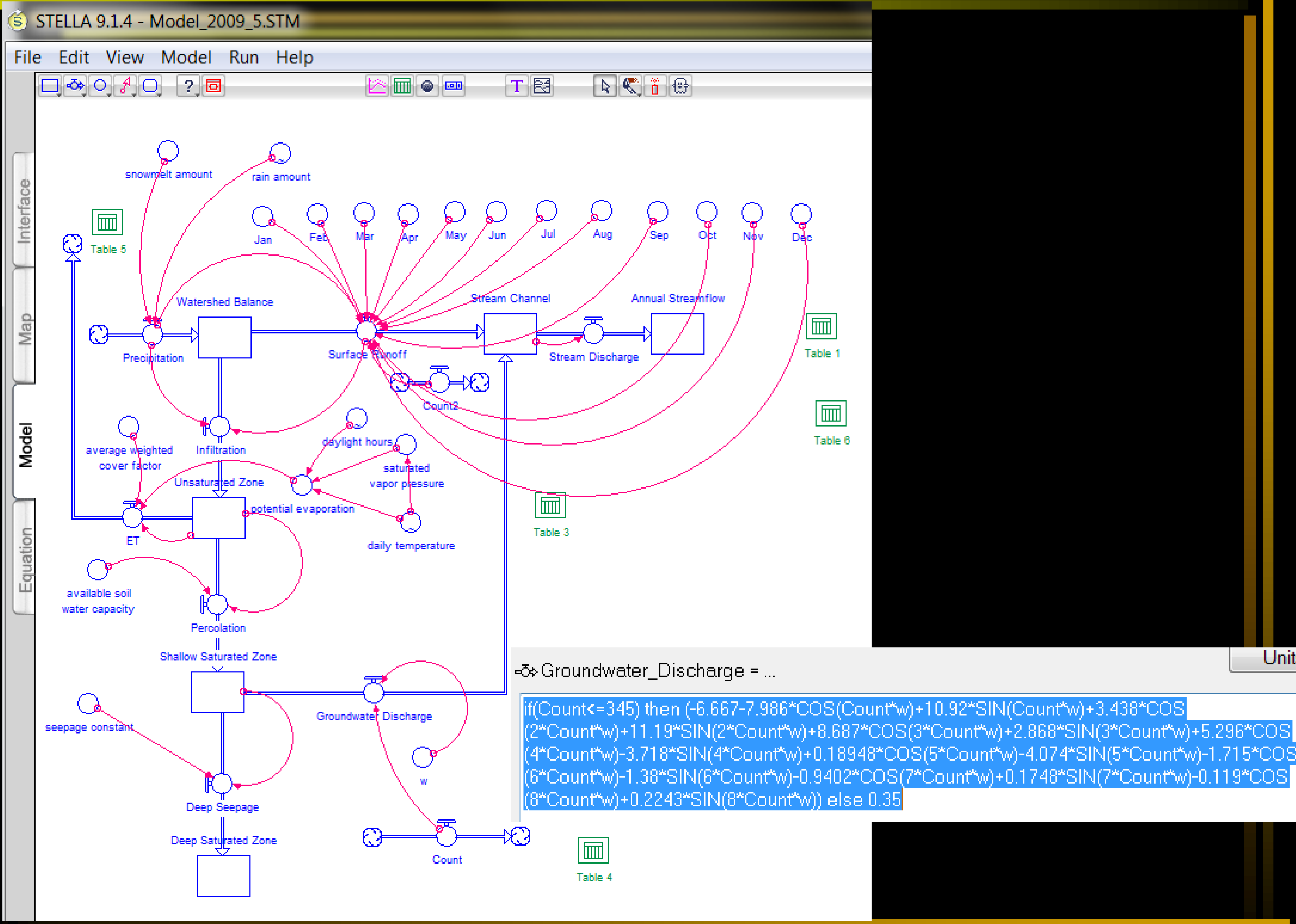


Sigma = 0.534 pixels

	X	Y	Row	Col	Active	DRow	DCol
1	1021977.519	1029420.228	2025	1603	True	0.20	0.68
2	1019244.673	1031508.275	1740	1007	True	-0.18	-0.07
3	1017052.571	1027117.995	2690	788	True	0.11	0.62
4	1019689.989	1026335.030	2725	1309	True	-0.25	-0.48
5	1020121.107	1026589.768	2657	1378	True	0.30	-0.55
6	1020574.593	1027030.874	2551	1443	True	0.05	-0.85
7	1021102.101	1026672.192	2597	1558	True	-0.28	1.13
8	1020672.028	1027132.216	2527	1458	True	0.04	0.40
9	1021226.837	1027079.295	2513	1563	True	0.31	0.32
10	1020811.563	1027408.465	2467	1471	True	-0.13	-0.73
11	1021353.837	1027567.182	2412	1566	True	-0.31	0.52
12	1021263.738	1028509.926	2233	1508	True	-0.19	-0.72
13	1020997.826	1029031.330	2144	1437	True	0.32	-0.27



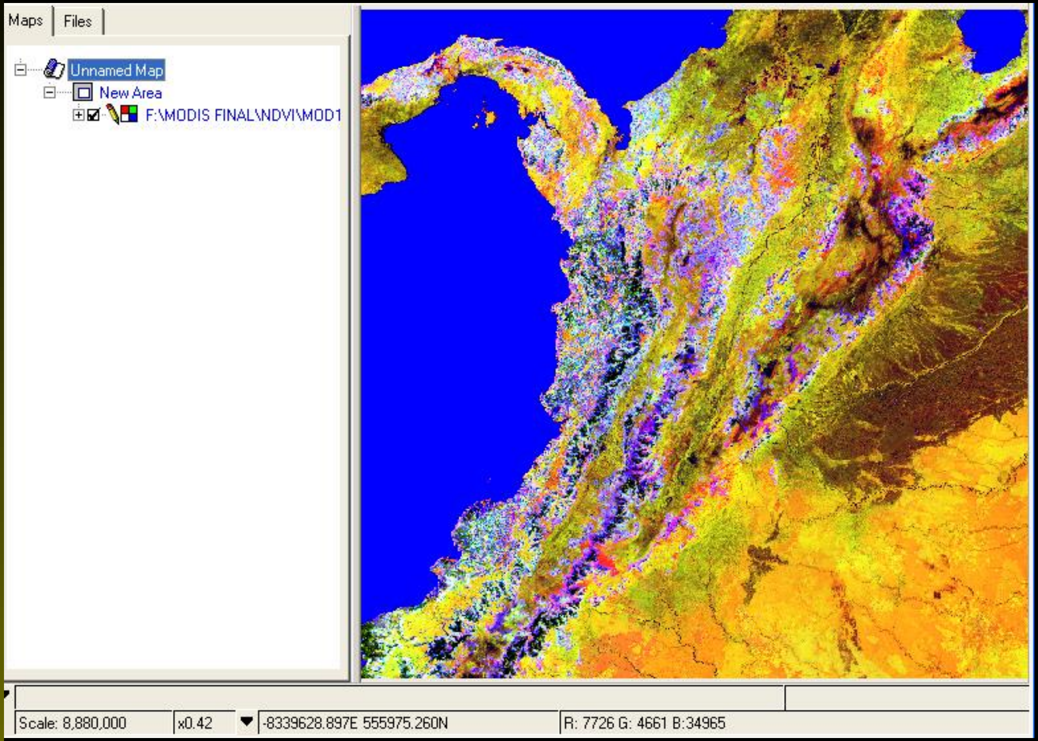

DIVER



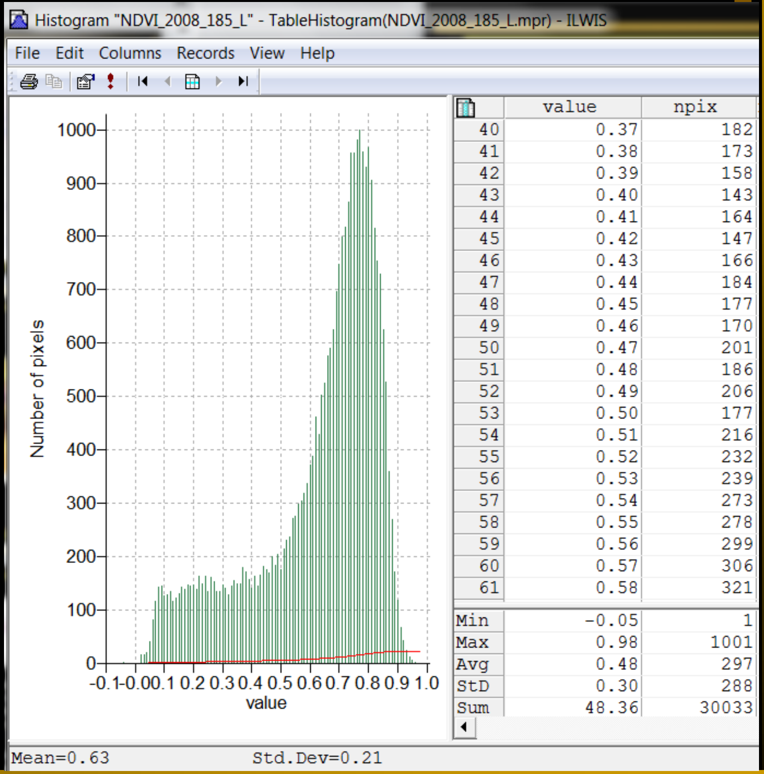
Groundwater_Discharge = ...

```
if(Count<=345) then (-6.667-7.986*COS(Count*w)+10.92*SIN(Count*w)+3.438*COS
(2*Count*w)+11.19*SIN(2*Count*w)+8.687*COS(3*Count*w)+2.868*SIN(3*Count*w)+5.296*COS
(4*Count*w)-3.718*SIN(4*Count*w)+0.18948*COS(5*Count*w)-4.074*SIN(5*Count*w)-1.715*COS
(6*Count*w)-1.38*SIN(6*Count*w)-0.9402*COS(7*Count*w)+0.1748*SIN(7*Count*w)-0.119*COS
(8*Count*w)+0.2243*SIN(8*Count*w)) else 0.35
```

NDVI



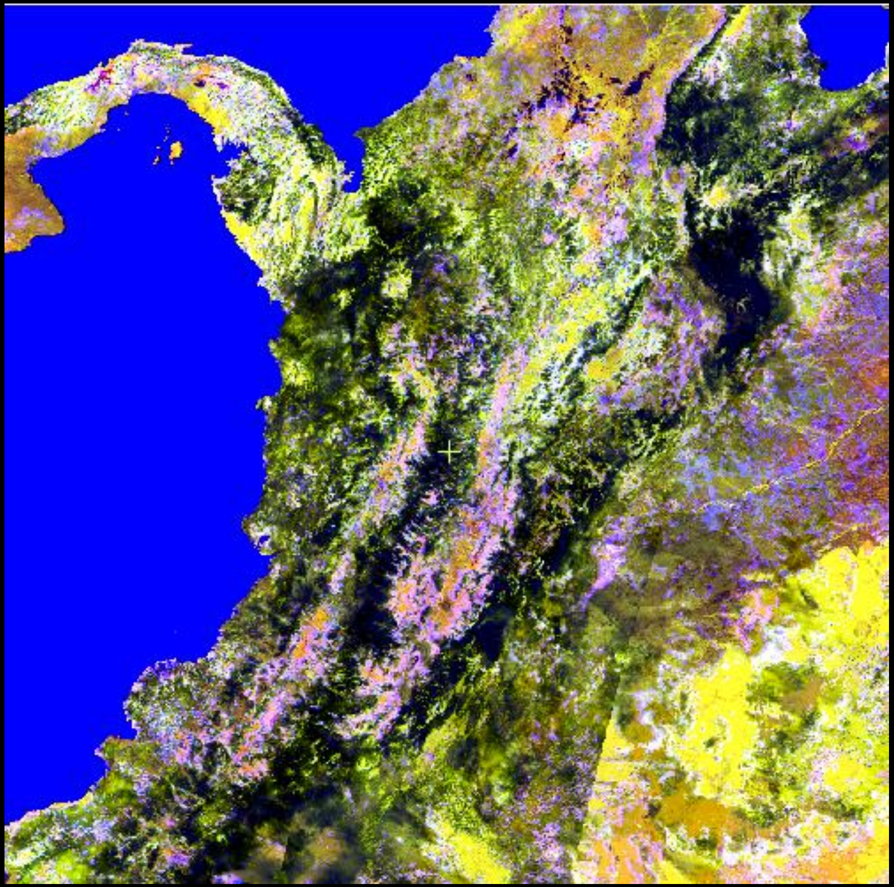
Scale: 8,880,000 | x0.42 | -8339628.897E 555975.260N | R: 7726 G: 4661 B:34965

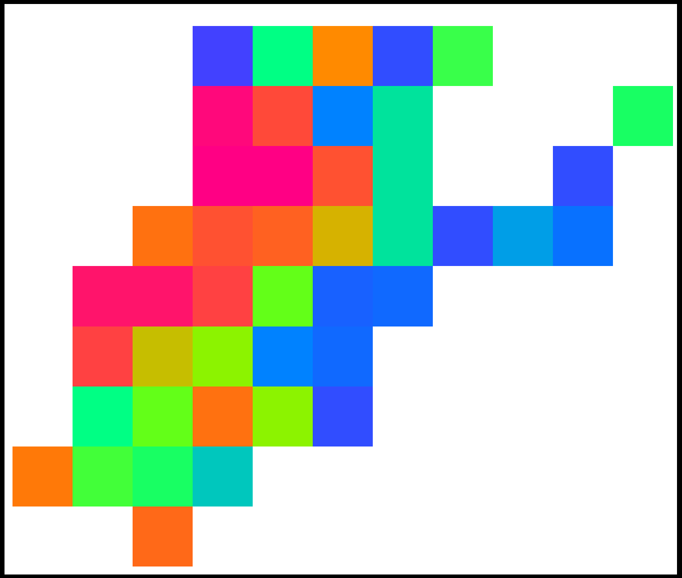

Histogram "NDVI_2008_185_L" - TableHistogram(NDVI_2008_185_L.mpr) - ILWIS

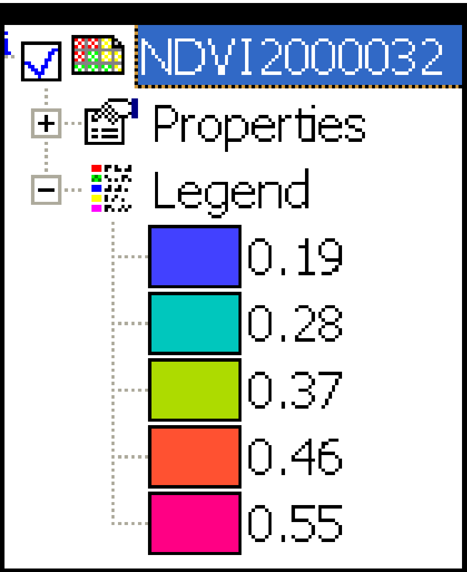
	value	npix
40	0.37	182
41	0.38	173
42	0.39	158
43	0.40	143
44	0.41	164
45	0.42	147
46	0.43	166
47	0.44	184
48	0.45	177
49	0.46	170
50	0.47	201
51	0.48	186
52	0.49	206
53	0.50	177
54	0.51	216
55	0.52	232
56	0.53	239
57	0.54	273
58	0.55	278
59	0.56	299
60	0.57	306
61	0.58	321
Min	-0.05	1
Max	0.98	1001
Avg	0.48	297
StD	0.30	288
Sum	48.36	30033

Mean=0.63 Std.Dev=0.21

TEMPORALIDAD MODIS







☑ NDVI2000032

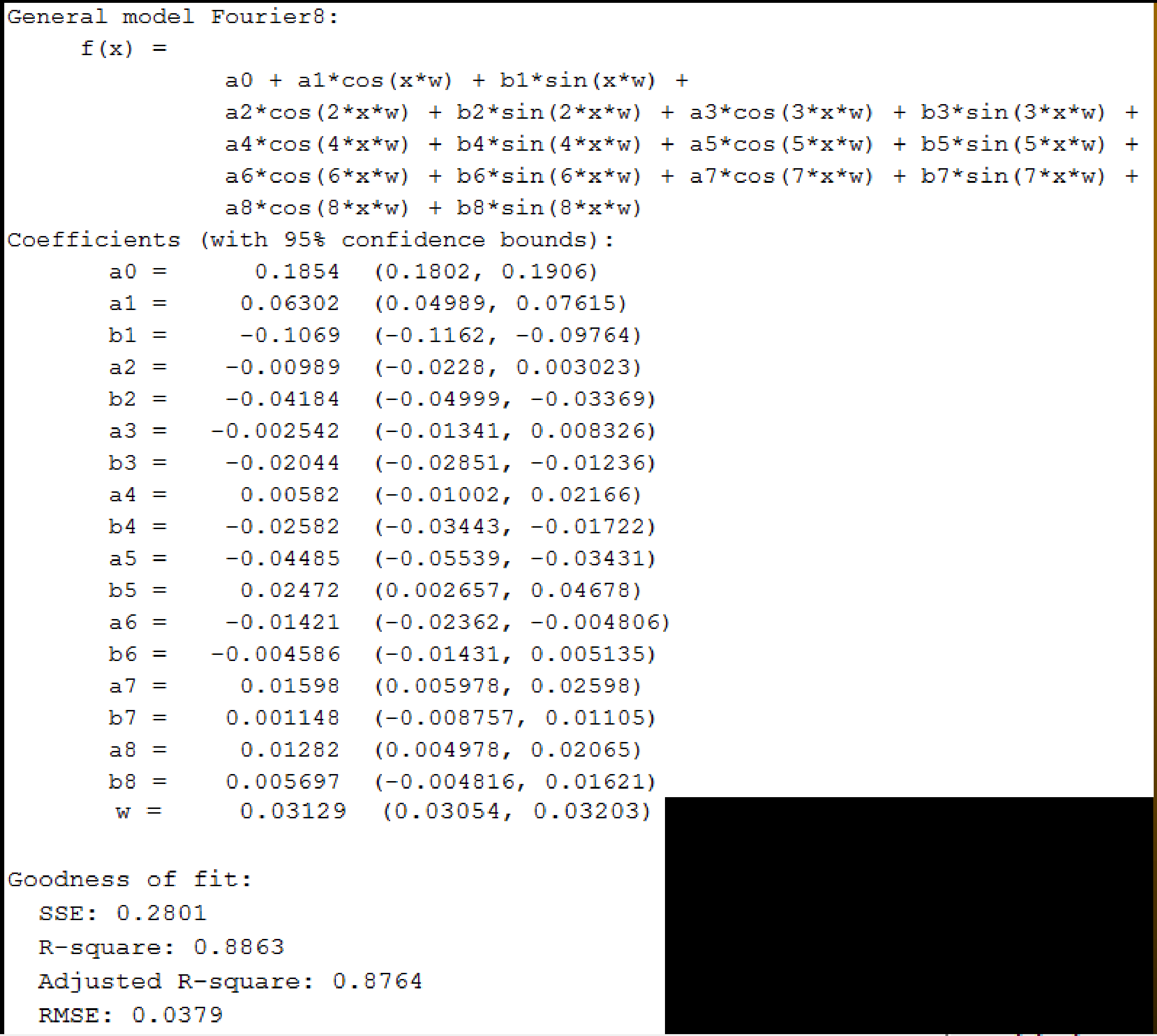
⊞ Properties

⊟ Legend

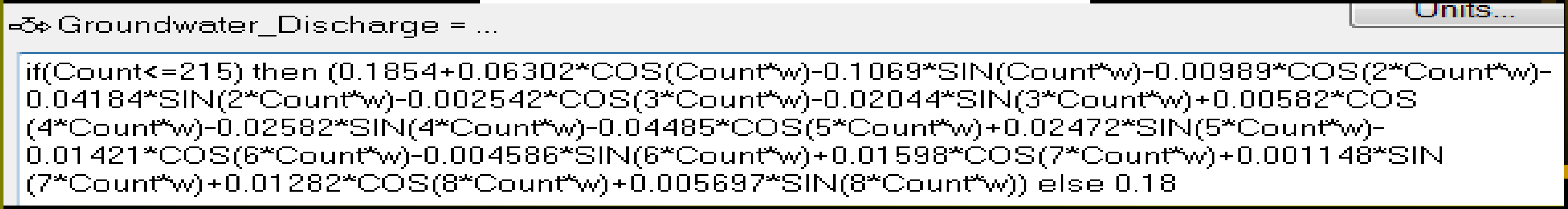
- 🟦 0.19
- 🟦 0.28
- 🟩 0.37
- 🟧 0.46
- 🟪 0.55

RESULTS

For
2008

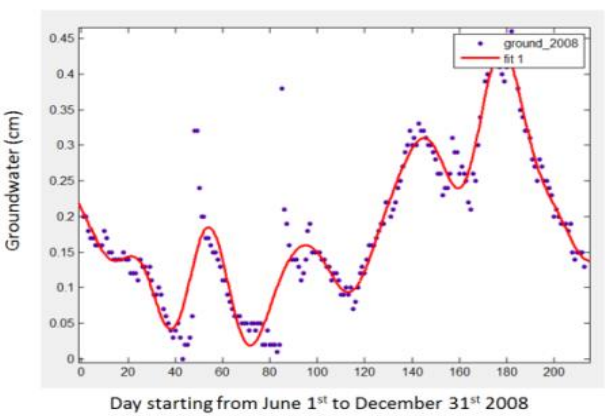


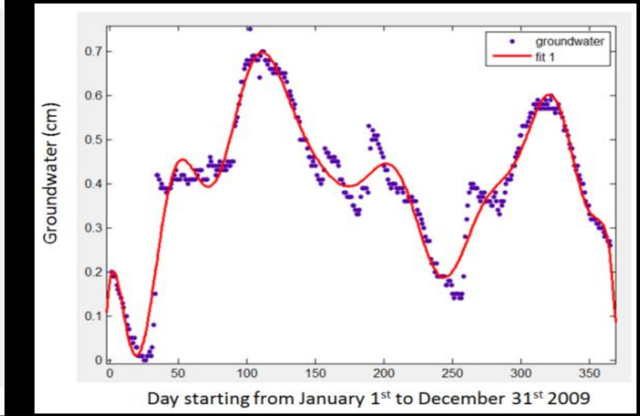
```
General model Fourier8:
     f(x) =
               a0 + a1*cos(x*w) + b1*sin(x*w) +
               a2*cos(2*x*w) + b2*sin(2*x*w) + a3*cos(3*x*w) + b3*sin(3*x*w) +
               a4*cos(4*x*w) + b4*sin(4*x*w) + a5*cos(5*x*w) + b5*sin(5*x*w) +
               a6*cos(6*x*w) + b6*sin(6*x*w) + a7*cos(7*x*w) + b7*sin(7*x*w) +
               a8*cos(8*x*w) + b8*sin(8*x*w)
Coefficients (with 95% confidence bounds):
       a0 =      0.1854  (0.1802, 0.1906)
       a1 =     0.06302  (0.04989, 0.07615)
       b1 =     -0.1069  (-0.1162, -0.09764)
       a2 =    -0.00989  (-0.0228, 0.003023)
       b2 =    -0.04184  (-0.04999, -0.03369)
       a3 =   -0.002542  (-0.01341, 0.008326)
       b3 =    -0.02044  (-0.02851, -0.01236)
       a4 =     0.00582  (-0.01002, 0.02166)
       b4 =    -0.02582  (-0.03443, -0.01722)
       a5 =    -0.04485  (-0.05539, -0.03431)
       b5 =     0.02472  (0.002657, 0.04678)
       a6 =    -0.01421  (-0.02362, -0.004806)
       b6 =   -0.004586  (-0.01431, 0.005135)
       a7 =     0.01598  (0.005978, 0.02598)
       b7 =    0.001148  (-0.008757, 0.01105)
       a8 =     0.01282  (0.004978, 0.02065)
       b8 =    0.005697  (-0.004816, 0.01621)
        w =     0.03129  (0.03054, 0.03203)

Goodness of fit:
  SSE: 0.2801
  R-square: 0.8863
  Adjusted R-square: 0.8764
  RMSE: 0.0379
```

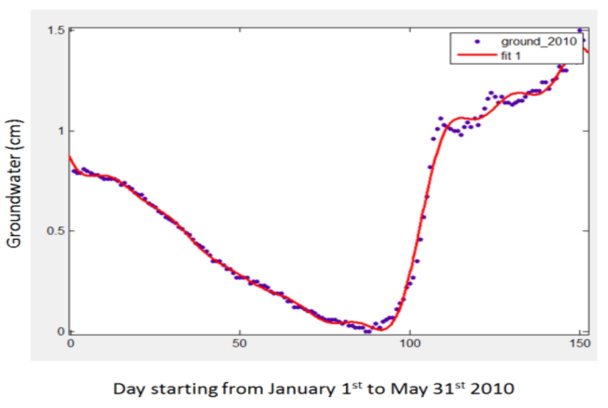


```
Groundwater_Discharge = ...
```

```
if(Count<=215) then (0.1854+0.06302*COS(Count*w)-0.1069*SIN(Count*w)-0.00989*COS(2*Count*w)-
0.04184*SIN(2*Count*w)-0.002542*COS(3*Count*w)-0.02044*SIN(3*Count*w)+0.00582*COS
(4*Count*w)-0.02582*SIN(4*Count*w)-0.04485*COS(5*Count*w)+0.02472*SIN(5*Count*w)-
0.01421*COS(6*Count*w)-0.004586*SIN(6*Count*w)+0.01598*COS(7*Count*w)+0.001148*SIN
(7*Count*w)+0.01282*COS(8*Count*w)+0.005697*SIN(8*Count*w)) else 0.18
```







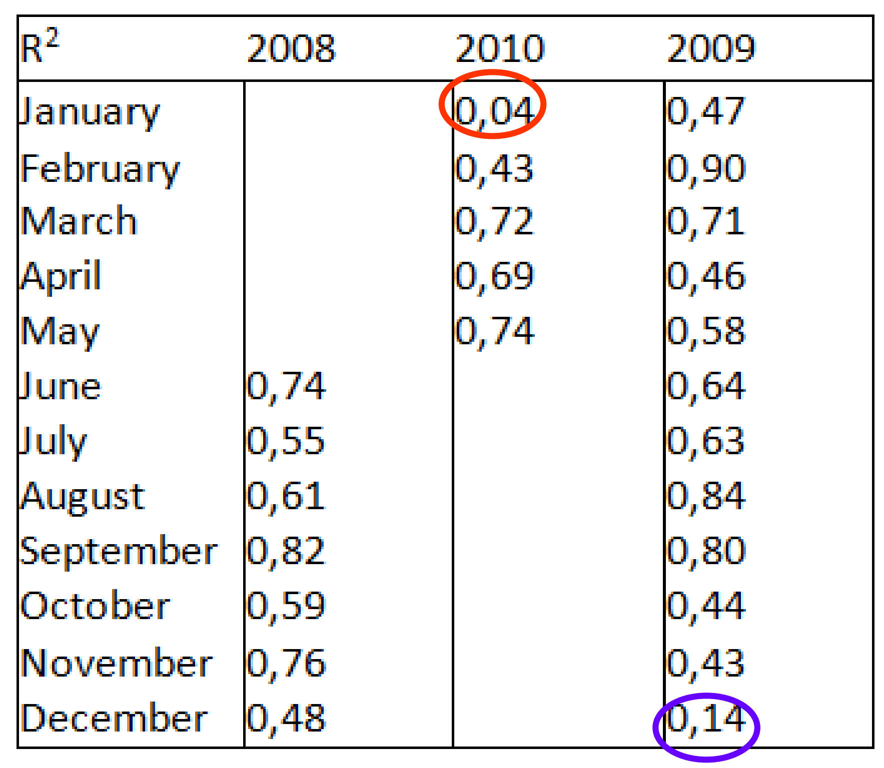
The groundwater data was transferred to the Stella Model by finding the best fitting, all of which were Fourier transformations done in Matlab. The discharge results from the STELLA model were compared to the measured values.

The groundwater measured by the piezometers and the calculated by Fourier transformations had fittings of 0.89 R^2 for the year 2008, 0.94 for 2009 and 0.99 for 2010

Year	Measured Average Peak Discharge m^3s^{-1}	Calculated Average Peak Discharge m^3s^{-1}	Period
2008	5.00	4.27	June-December
2009	5.01	4.94	January-December
2010	3.08	3.04	January-May

Month	Rain (mm) 2008	Rain (mm) 2009	Rain (mm) 2010
January	32.90	31.60	0
February	23.00	18.90	9.3
March	32.60	50.40	19.0
April	48.10	35.60	100.2
May	94.60	47.80	61.9
June	84.10	28.20	
July	72.60	53.10	
August	58.80	52.60	
September	12.80	29.80	
October	90.60	52.10	
November	145.90	46.20	
December	26.70	1.80	

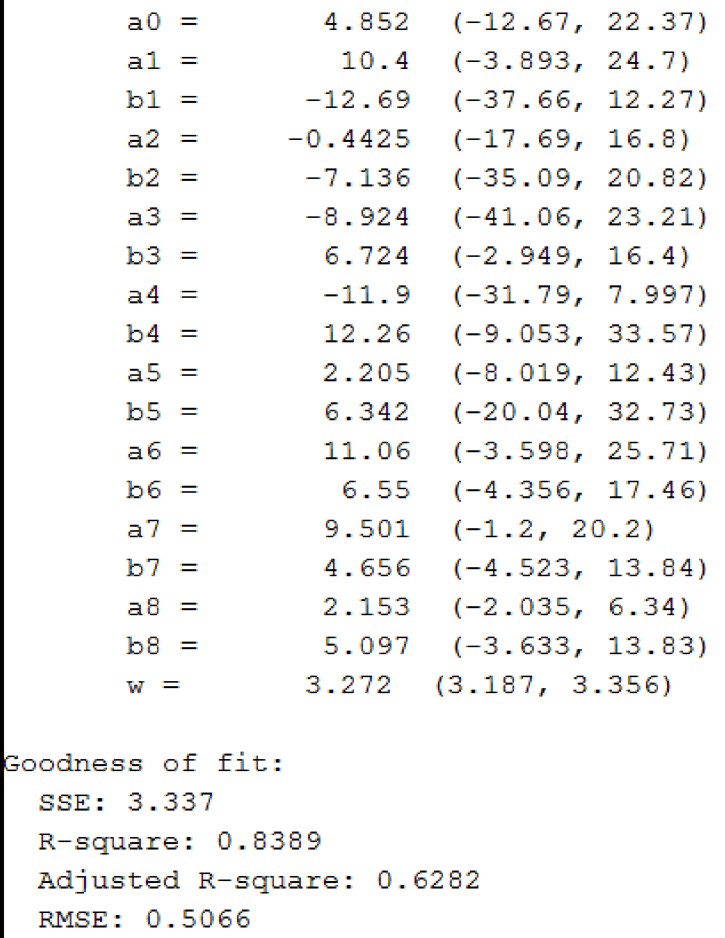
Discharge fittings were also performed per monthly basis. All of which were Fourier transformations



R^2	2008	2010	2009
January		0,04	0,47
February		0,43	0,90
March		0,72	0,71
April		0,69	0,46
May		0,74	0,58
June	0,74		0,64
July	0,55		0,63
August	0,61		0,84
September	0,82		0,80
October	0,59		0,44
November	0,76		0,43
December	0,48		0,14

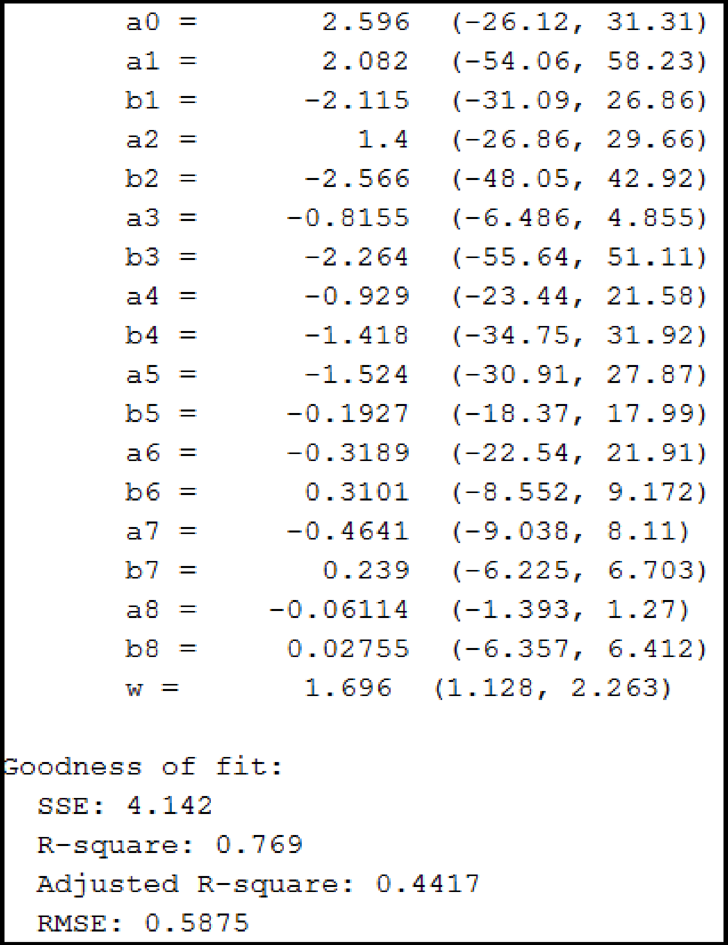
Month	Rain (mm) 2008	Rain (mm) 2009	Rain (mm) 2010
January	32.90	31.60	0
February	23.00	18.90	9.3
March	32.60	50.40	19.0
April	48.10	35.60	100.2
May	94.60	47.80	61.9
June	84.10	28.20	
July	72.60	53.10	
August	58.80	52.60	
September	12.80	29.80	
October	90.60	52.10	
November	145.90	46.20	
December	26.70	1.80	

August 2009



```
       a0 =       4.852  (-12.67, 22.37)
       a1 =        10.4  (-3.893, 24.7)
       b1 =      -12.69  (-37.66, 12.27)
       a2 =     -0.4425  (-17.69, 16.8)
       b2 =      -7.136  (-35.09, 20.82)
       a3 =      -8.924  (-41.06, 23.21)
       b3 =       6.724  (-2.949, 16.4)
       a4 =       -11.9  (-31.79, 7.997)
       b4 =       12.26  (-9.053, 33.57)
       a5 =       2.205  (-8.019, 12.43)
       b5 =       6.342  (-20.04, 32.73)
       a6 =       11.06  (-3.598, 25.71)
       b6 =        6.55  (-4.356, 17.46)
       a7 =       9.501  (-1.2, 20.2)
       b7 =       4.656  (-4.523, 13.84)
       a8 =       2.153  (-2.035, 6.34)
       b8 =       5.097  (-3.633, 13.83)
       w =       3.272  (3.187, 3.356)

Goodness of fit:
  SSE: 3.337
  R-square: 0.8389
  Adjusted R-square: 0.6282
  RMSE: 0.5066
```

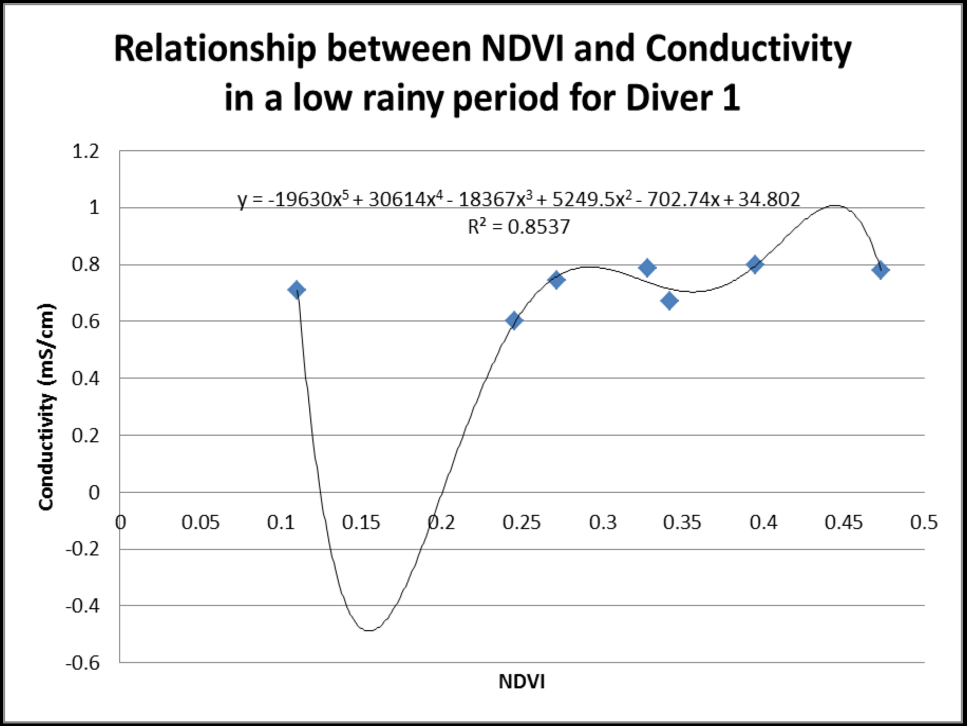
November 2008



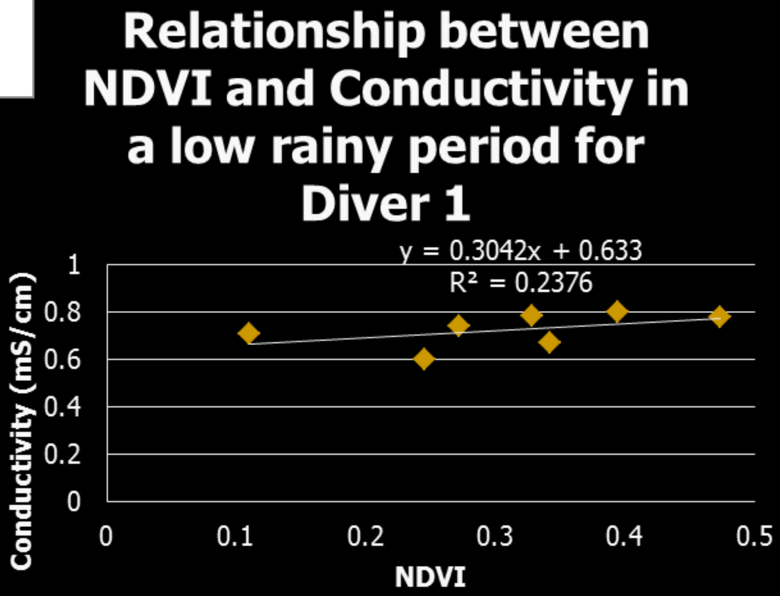
```
       a0 =       2.596  (-26.12, 31.31)
       a1 =       2.082  (-54.06, 58.23)
       b1 =      -2.115  (-31.09, 26.86)
       a2 =         1.4  (-26.86, 29.66)
       b2 =      -2.566  (-48.05, 42.92)
       a3 =     -0.8155  (-6.486, 4.855)
       b3 =      -2.264  (-55.64, 51.11)
       a4 =      -0.929  (-23.44, 21.58)
       b4 =      -1.418  (-34.75, 31.92)
       a5 =      -1.524  (-30.91, 27.87)
       b5 =     -0.1927  (-18.37, 17.99)
       a6 =     -0.3189  (-22.54, 21.91)
       b6 =      0.3101  (-8.552, 9.172)
       a7 =     -0.4641  (-9.038, 8.11)
       b7 =       0.239  (-6.225, 6.703)
       a8 =    -0.06114  (-1.393, 1.27)
       b8 =     0.02755  (-6.357, 6.412)
       w =       1.696  (1.128, 2.263)

Goodness of fit:
  SSE: 4.142
  R-square: 0.769
  Adjusted R-square: 0.4417
  RMSE: 0.5875
```

Julian Day	Rain	Conductivity	Conductivity	NDVI
	Sum	Average	Average	
		Diver 1	Diver 2	
	mm	mS/cm	mS/cm	
129	46.65	0.6	0.76	0.25
145	64.96	0.64	0.82	0.10
161	35.49	0.67	0.91	0.34
177	62.70	0.69	1.04	0.30
193	21.02	0.71	1.11	0.11
209	71.56	0.73	1.16	0.14
225	19.01	0.74	1.18	0.27
241	67.75	0.76	1.21	0.23
257	35.26	0.77	1.23	0.48
273	19.02	0.78	1.25	0.47
289	32.96	0.79	1.27	0.33
305	20.06	0.80	1.29	0.39
321	89.64	0.81	1.30	0.41

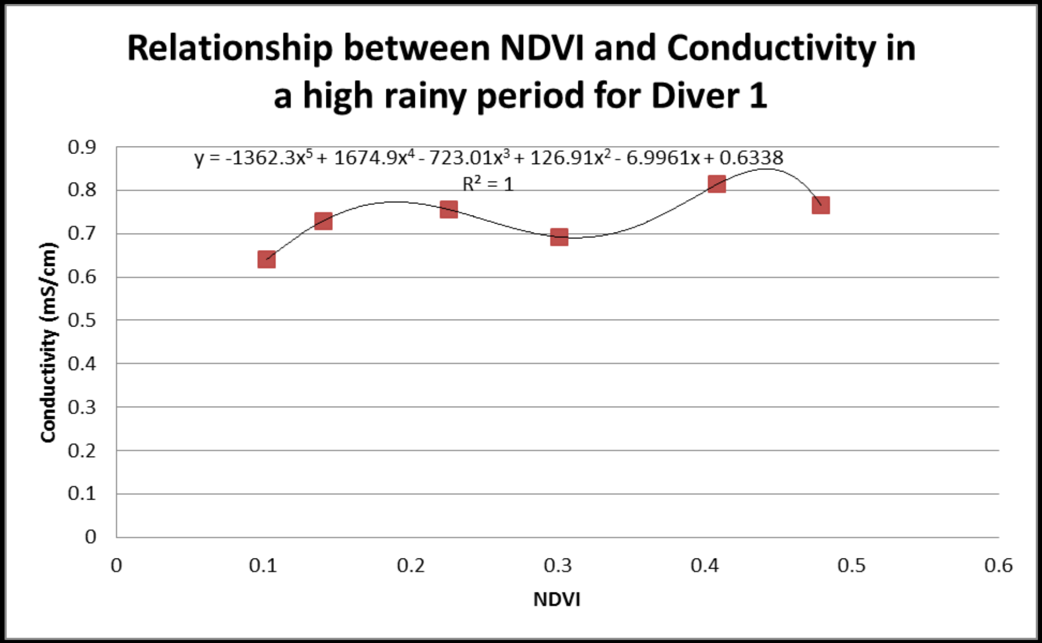
Zone 1

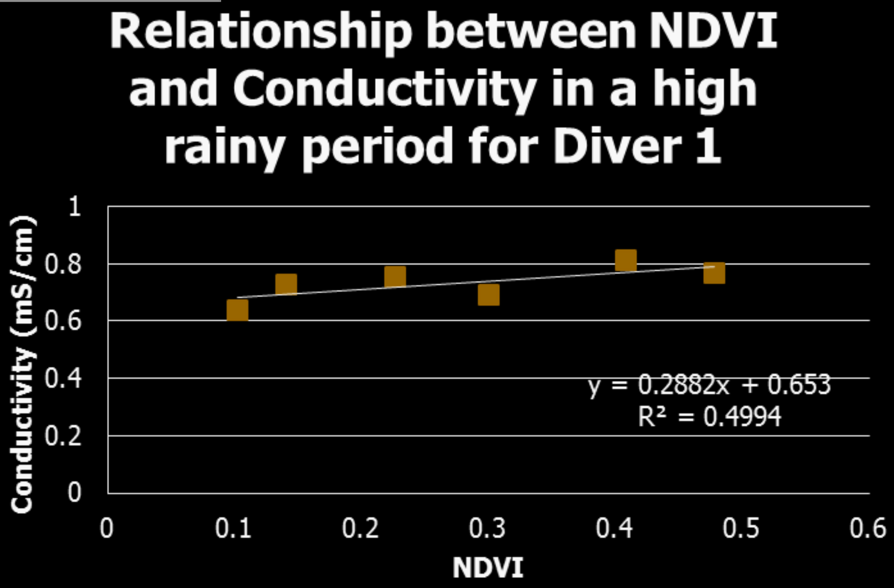


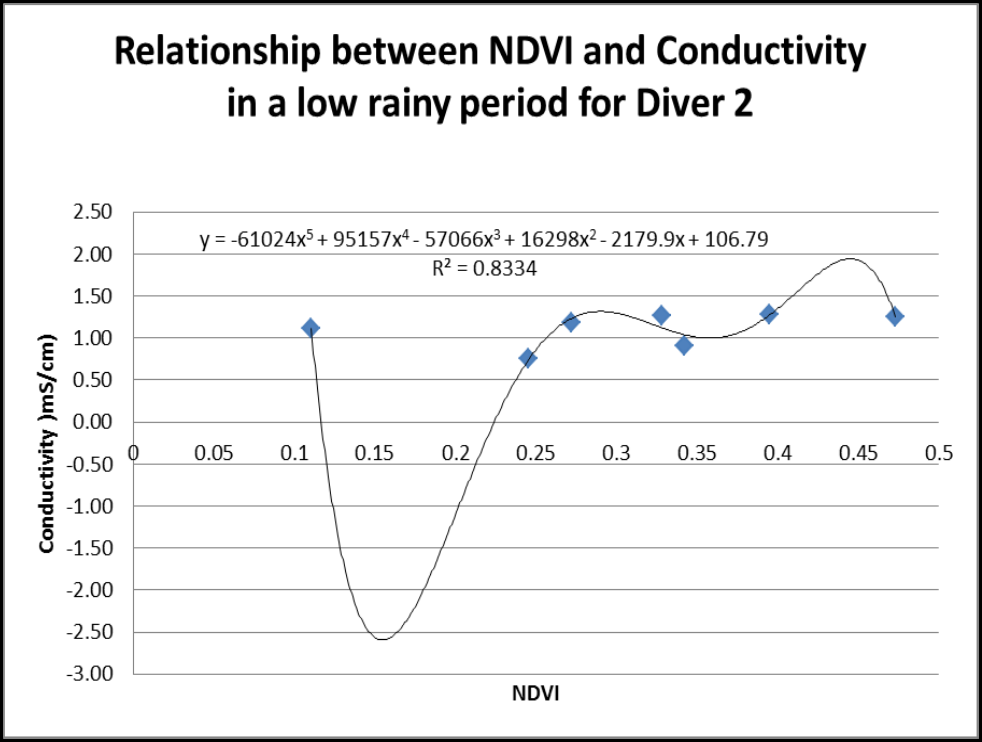
Relationship between NDVI and Conductivity in a low rainy period for Diver 1



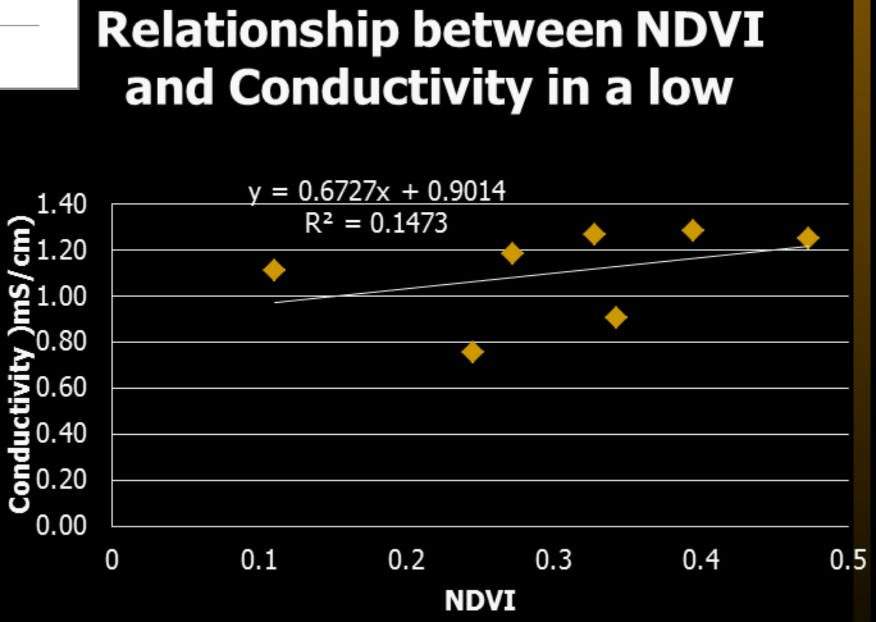
Relationship between NDVI and Conductivity in a low rainy period for Diver 1

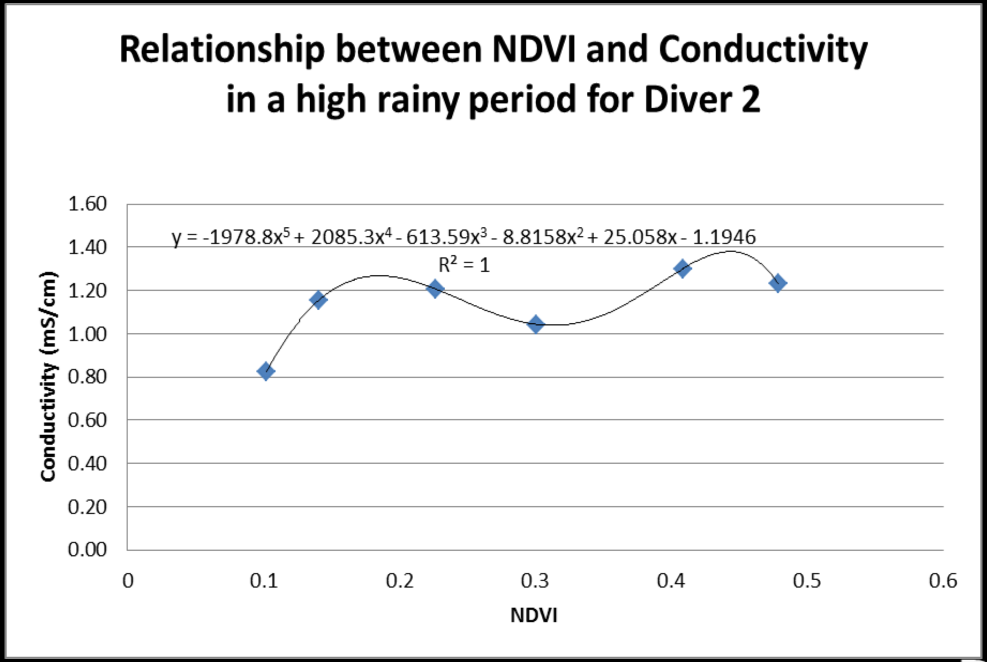

Relationship between NDVI and Conductivity in a high rainy period for Diver 1
y = -1362.3x⁵ + 1674.9x⁴ - 723.01x³ + 126.91x² - 6.9961x + 0.6338
R² = 1
Conductivity (mS/cm)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
NDVI


Relationship between NDVI and Conductivity in a high rainy period for Diver 1
Conductivity (mS/cm)
1
0.8
0.6
0.4
0.2
0
y = 0.2882x + 0.653
R² = 0.4994
0
0.1
0.2
0.3
0.4
0.5
0.6
NDVI


Relationship between NDVI and Conductivity in a low rainy period for Diver 2
$y = -61024x^5 + 95157x^4 - 57066x^3 + 16298x^2 - 2179.9x + 106.79$
$R^2 = 0.8334$
Conductivity)mS/cm)
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
NDVI

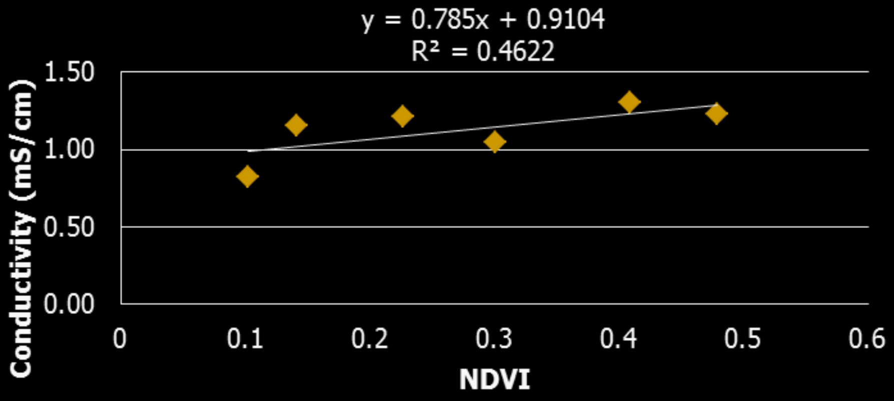
Zone 3


Relationship between NDVI and Conductivity in a low
$y = 0.6727x + 0.9014$
$R^2 = 0.1473$
Conductivity)mS/cm)
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0
0.1
0.2
0.3
0.4
0.5
NDVI



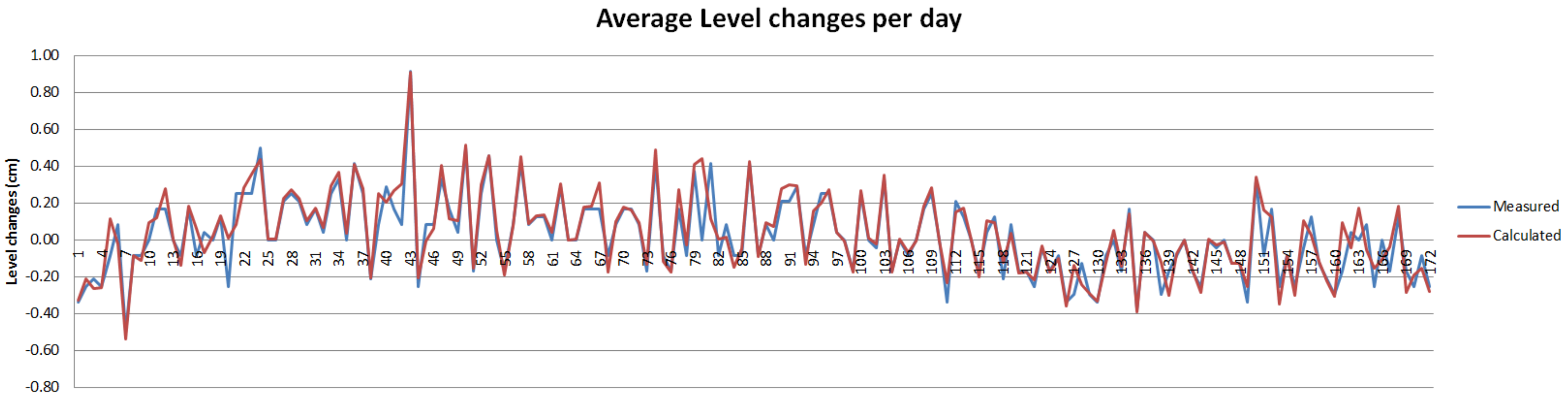
Relationship between NDVI and Conductivity in a high rainy period for Diver 2

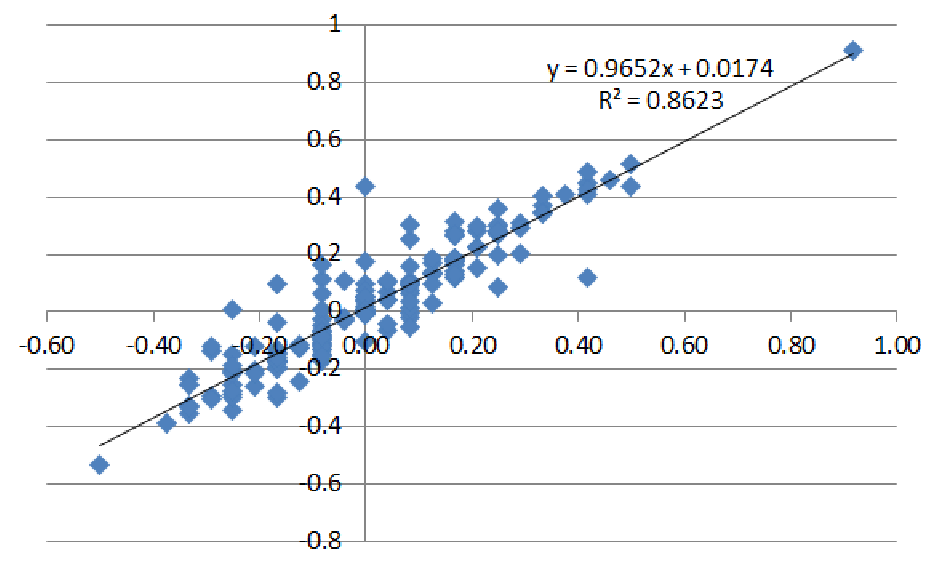
Relationship between NDVI and Conductivity in a high rainy period for Diver 2



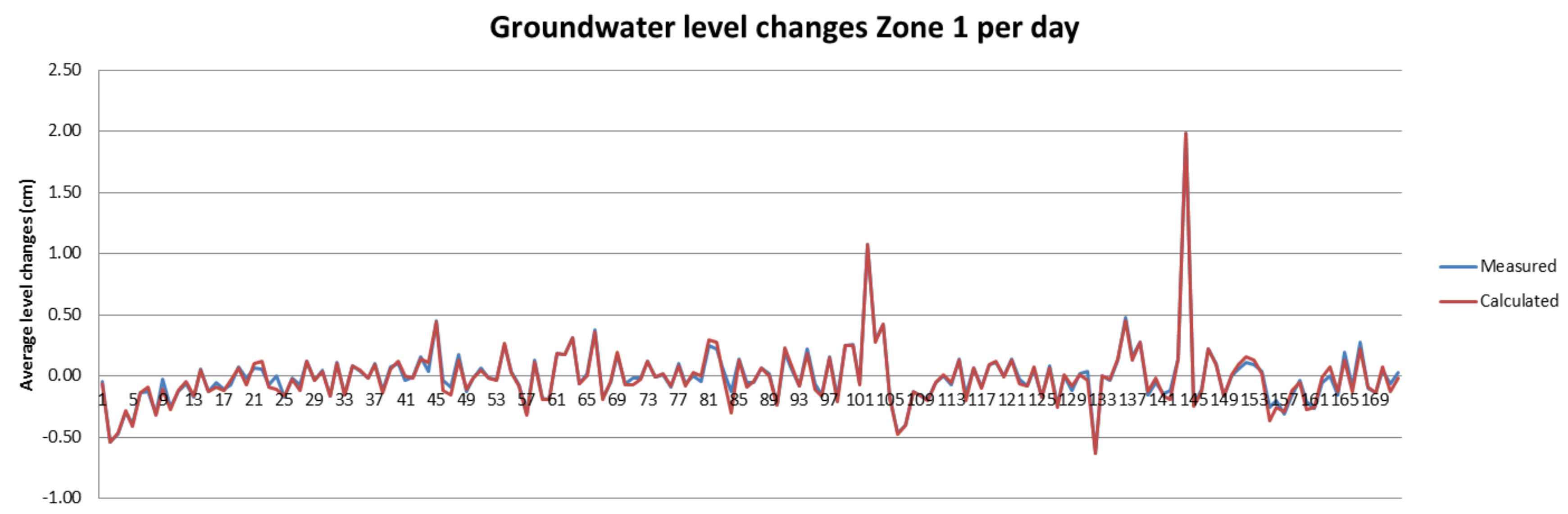
With respect to the average level changes per day and rain, the relationship is as follows for the studied Zone 3.

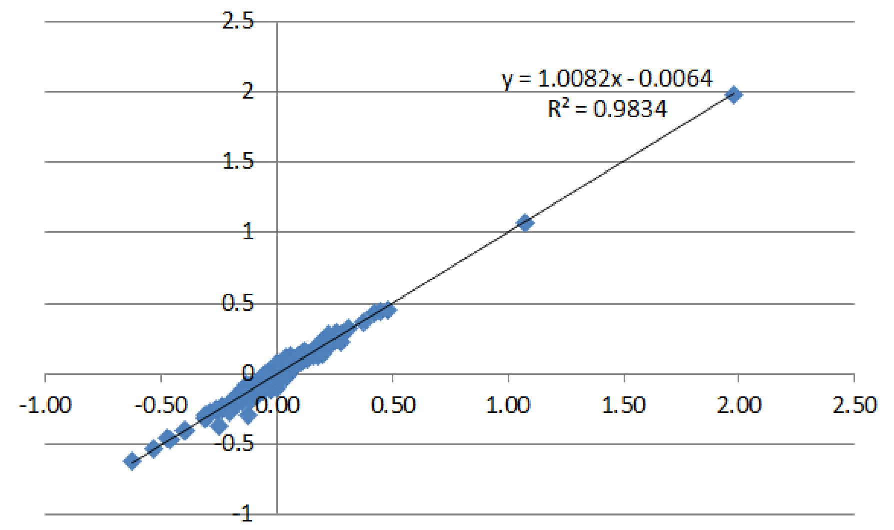
$$\hat{Y}_{t+1} = 0.0061 + 1.06\,Y_{t-1} + 0.3578\,\text{Rain}_{t} - 0.2433\,\text{Rain}_{t-1} + 0.0209\,\text{Rain}_{t-2} - 0.0058\,\text{Rain}_{t-3} - 0.0499\,\text{Rain}_{t-4}$$





$$\hat{Y}_{t+1} = -0.0041 + Y_{t-1} + 0.01\,\text{Rain}_{t} - 0.05\,\text{Rain}_{t-1} + 0.08\,\text{Rain}_{t-2} - 0.0229\,\text{Rain}_{t-3} - 0.1372\,\text{Rain}_{t-4}$$





CONCLUSIONS

The groundwater level changes measured by the divers and the calculated values under Fourier transformations had fittings of 0.89 R^2 for the year 2008, 0.94 for 2009 and 0.99 for 2010.

Those transformations were used in the Stella Model, determining measured discharge peaks compared with calculated discharge peaks very similar to each other.

For example for 2008 the measured average discharge peak was 5 m^3s^{-1}, while the calculated was 4.27 m^3s^{-1} , for 2009 the measured average discharge peak value was 5.01 m^3s^{-1} as compared to 4.94 m^3s^{-1} calculated, and 3.08 m^3s^{-1} measured average peak value for 2010 versus 3.04 m^3s^{-1} calculated.

With respect to the fittings of calculated versus measured discharges by months, most of them had fittings with R^2 over 0.43 up to 0.9, except January 2010 and December 2009, months that had the lowest rain scenario (0 mm and 1.8 mm respectively). The results show that the model does not perform effectively for very low rain values.

- There results also show a relationship between NDVI and groundwater conductivity, such relationship varies also depending upon the rainy period. With polynomial fittings at low rainy periods the corresponding R^2 was 0.83 and almost 0.86 for Zones 3 and 1 respectively. For high rainy periods in both cases the R^2 was 1. Reinforcing the fact that dry and rain seasons interfere in the results.
- It was also found that time series can relate the groundwater level changes with rain, specially important when divers are not easy to place. The R^2 value for the measured and predicted values in two study zones was higher than 0.86 for one scenario and 0.98 for the other.

Thank You

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