

# *MODELLING SPATIOTEMPORAL VARIABILITY OF WATER TABLE DEPTHS MONITORING DATA*



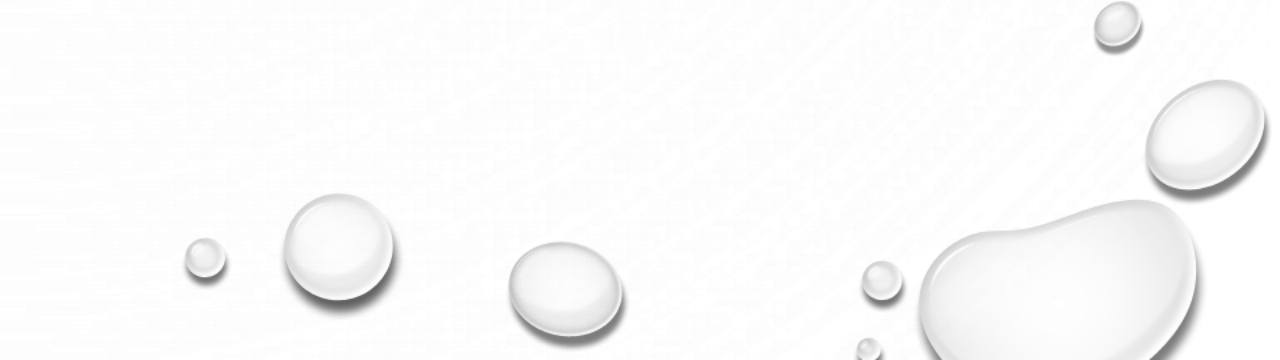
PROF. DR. RODRIGO LILLA MANZIONE

UNESP – SÃO PAULO STATE UNIVERSITY

CAMPUS OF OURINHOS



# OUTLINE

- GEOSTATISTICAL APPROACHES FOR SPATIO-TEMPORAL DATA
  - COVARIANCE MODELS
  - MONITORING NETWORK FOR WATER TABLE DEPTHS
  - PRELIMINARY RESULTS
  - CONCLUSIONS
  - FUTURE STUDIES
- 

# INTRODUCTION

GROUNDWATER MONITORING DATA ARE PARTICULARLY INTERESTING WHEN ANALYSING AQUIFER CHARACTERISTICS BECAUSE CAN REVEAL NOT ONLY TEMPORAL PATTERNS BUT ALSO SPATIAL DISTRIBUTIONS AND VARIATION OVER TIME IF COLLECTED IN A GEOSPATIAL NETWORK.

FROM SPATIAL AND TEMPORAL CORRELATIONS, IT IS POSSIBLE TO PREDICT VALUES AT POINTS FROM NEIGHBOURING OBSERVATIONS AND MAKE PREDICTIONS IN BETWEEN OBSERVATION TIMES.

SPATIO-TEMPORAL INTERPOLATION CAN POTENTIALLY PROVIDE MORE ACCURATE PREDICTIONS THAN SPATIAL INTERPOLATION BECAUSE OBSERVATIONS TAKEN AT OTHER TIMES CAN BE INCLUDED.

# SPATIO-TEMPORAL GEOSTATISTICS

KYRIAKIDIS & JOURNEL (1999) DEFINE TWO MAJOR CONCEPTUAL VIEWPOINTS FOR MODELING OF SPATIOTEMPORAL DISTRIBUTIONS VIA SPATIAL STATISTICS TOOLS EXTENDED TO INCLUDE THE ADDITIONAL TIME DIMENSION:

A SINGLE SPATIOTEMPORAL RF MODEL  $Z(\mathbf{U}, T)$ , TYPICALLY DECOMPOSED INTO A TREND COMPONENT MODELING SOME “AVERAGE” SMOOTH VARIABILITY OF THE SPATIOTEMPORAL PROCESS  $Z(\mathbf{U}, T)$ , AND A STATIONARY RESIDUAL COMPONENT MODELING HIGHER FREQUENCY FLUCTUATIONS AROUND THAT TREND IN BOTH SPACE AND TIME.

MULTIPLE VECTORS OF RFS OR VECTORS OF TS. TWO MODEL SUBCLASSES CAN THEN BE DEFINED. MODELS IN THE FIRST SUBCLASS TREAT THE SPATIOTEMPORAL RF  $Z(\mathbf{U}, T)$  AS A COLLECTION OF A FINITE NUMBER  $T$  OF TEMPORALLY CORRELATED SPACE RFS  $\mathbf{z}(\mathbf{U})$ , WHILE MODELS IN THE SECOND SUBCLASS VIEW THE RF  $Z(\mathbf{U}, T)$  AS A COLLECTION OF A FINITE NUMBER  $N$  OF SPATIALLY CORRELATED TS  $\mathbf{z}(T)$ .

# SPATIO-TEMPORAL COVARIANCE MODELS

ESTIMATING AND MODELLING THE CORRELATION OF A SPACE-TIME PROCESS IS FUNDAMENTAL TO GEOSTATISTICAL ANALYSIS, SINCE ONLY IF THE CORRELATION MODEL IS APPROPRIATE FOR THE VARIABLE UNDER STUDY CAN THE SUBSEQUENT ESTIMATION AND/OR SIMULATION RESULTS BE RELIED ON (DE IACO, 2010).

IN PRACTICE IT IS CUSTOMARY TO PROPOSE A MODEL AND TEST ITS PERMISSIBILITY USING ONE OF A SET OF BASIC MODELS THAT ARE KNOWN TO BE PERMISSIBLE.

THAT IS A RELATIVELY STRAIGHTFORWARD PRACTICE IN MODELLING SPATIAL (OR TEMPORAL) VARIOGRAMS AS THERE ARE MANY SUCH MODELS IN COMMON USE AND THESE MODELS MAY BE COMBINED LINEARLY TO FORM COMPLEX MODELS.

# SPATIOTEMPORAL COVARIANCE MODELS

- SEPARABLE/LINEAR MODEL (ROUHANI & HALL, 1989)
- PRODUCT MODEL (DE CESARE ET AL., 2001)
- PRODUCT-SUM MODEL (DE IACO ET AL., 2001)
- METRIC MODEL (DIMITRAKOPOULOS & LUO, 1994; SOARES & PEREIRA; 2007).
- CRESSIE-HUANG MODEL (CRESSIE & HUANG, 1999),
- GNEITING MODELS (GNEITING, 2002),
- INTEGRATED PRODUCT MODEL AND MIXTURE MODELS (DE IACO ET AL., 2002A)
- INTEGRATED PRODUCT-SUM MODEL (DE IACO ET AL., 2002B)
- NON-SEPARABLE SPATIOTEMPORAL COVARIANCE MODELS (KOLOVOS ET AL., 2004; STEIN, 2005; PORCU ET AL., 2008; RODRIGUES & DIGGLE, 2010).

# COVARIANCE MODELS – NOTION OF SEPARABILITY

A *SEPARABLE* SPACE–TIME COVARIANCE FUNCTION CONSIDERS THE SPATIOTEMPORAL PROCESS AS THE JOINT PROCESS OF TWO INDEPENDENT PROCESSES, ONE THAT OCCURS IN SPACE AND ANOTHER THAT OCCURS IN TIME, RESULTING IN A PURELY SPATIAL COMPONENT AND A PURELY TEMPORAL COMPONENT.

HOWEVER, WE DO NOT OBSERVE REALISATIONS OF THE TWO SEPARATE PROCESSES, ONLY THE JOINT PROCESS.

SEPARABILITY IS RESTRICTIVE AND OFTEN REQUIRES UNREALISTIC ASSUMPTIONS.

# COVARIANCE MODELS

IN THIS STUDY WE TESTED THREE ST COVARIANCE MODELS:

SEPARABLE

PRODUCT-SUM

METRIC



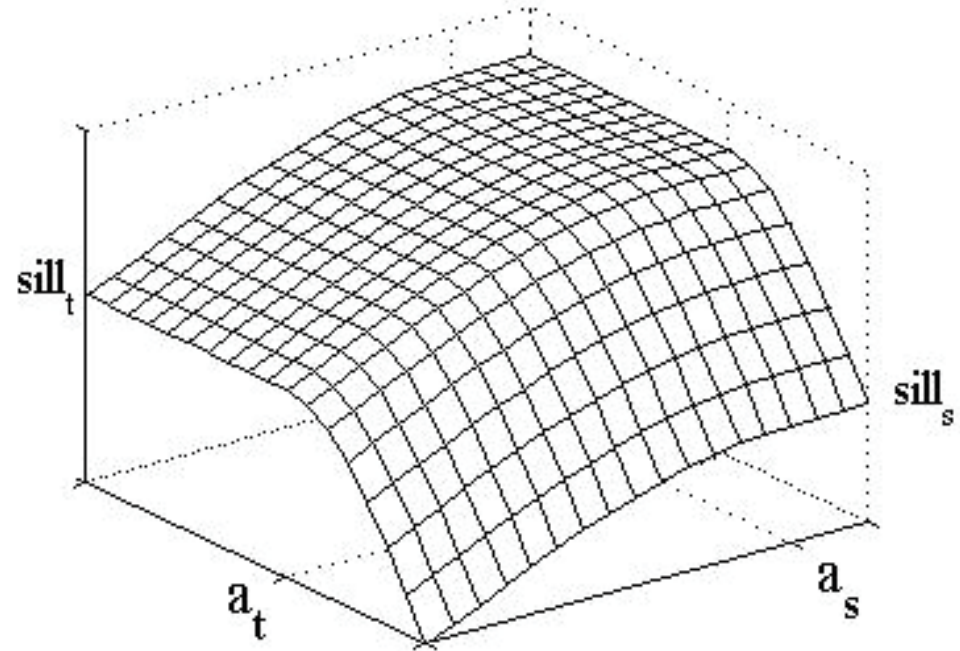
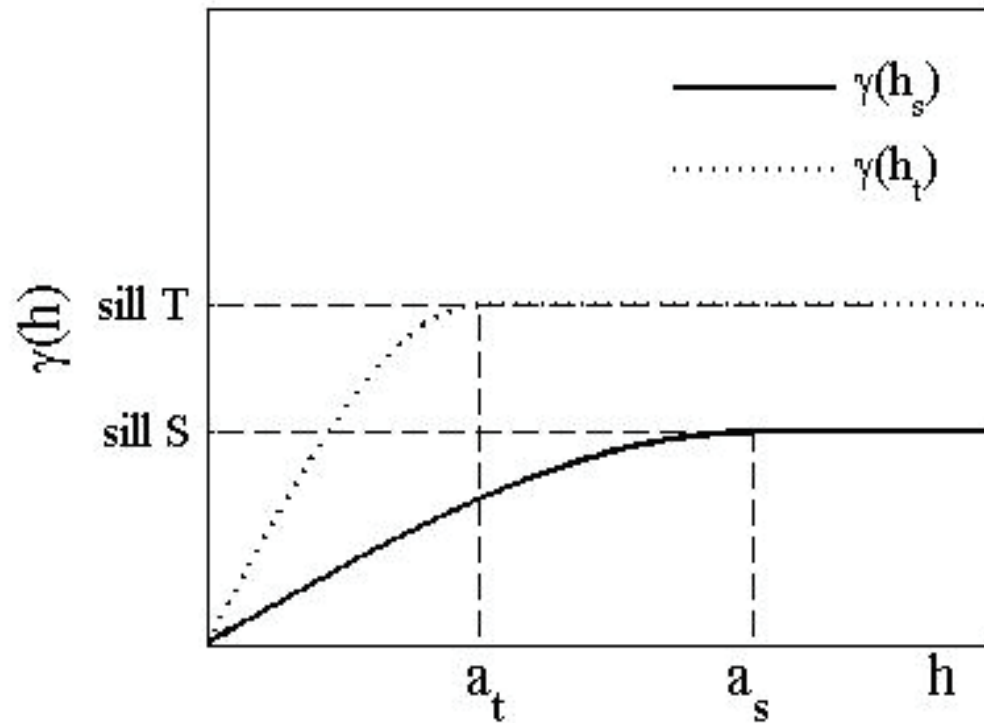
# SEPARABLE

$$C_{st}(h_s, h_t) = C_s(h_s) + C_t(h_t)$$

$C_s(h_s)$  SPATIAL COVARIANCE AND  $C_t(h_t)$  TEMPORAL COVARIANCE SIMPLE ADDED

FOR THIS MODEL, COVARIANCE MATRICES OF CERTAIN CONFIGURATIONS OF SPATIOTEMPORAL DATA ARE SINGULAR (MYERS & JOURNEL, 1990; ROUHANI & MYERS, 1990).

THIS MEANS THIS COVARIANCE FUNCTION IS ONLY POSITIVE SEMI-DEFINITE AND IS UNSATISFACTORY FOR OPTIMAL PREDICTION.



Example of separable model, semivariograms in (omnidirectional) spatial and temporal directions (left) and spatiotemporal semivariogram (right).

SOURCE: Denham (2012)

# PRODUCT-SUM

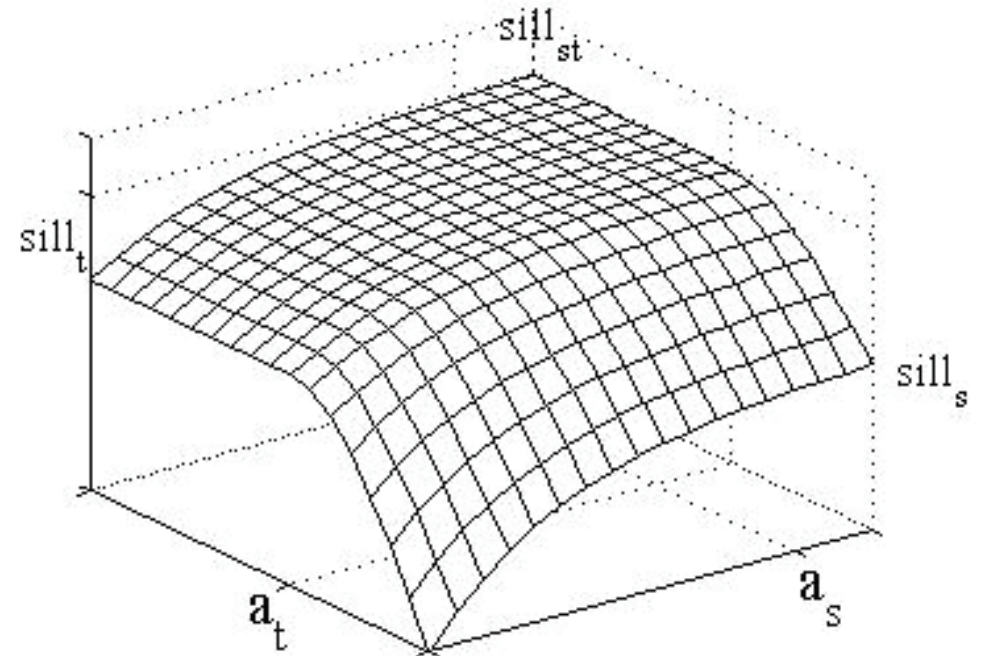
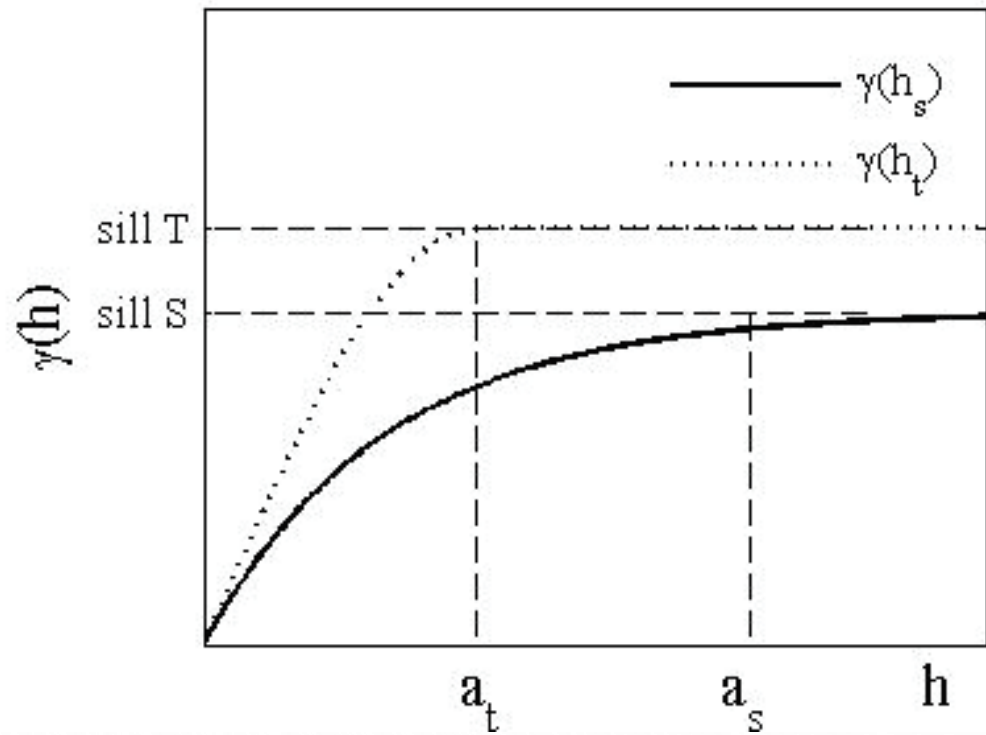
$$C_{st}(h_s; h_t) = k_1 C_s(h_s) C_t(h_t) + k_2 C_s(h_s) + k_3 C_t(h_t)$$

$$\gamma_{st}(h_s; h_t) = [k_2 + k_1 C_t(0)] \gamma_s(h_s) + [k_3 + k_1 C_s(0)] \gamma_t(h_t) - k_1 \gamma_s(h_s) \gamma_t(h_t)$$

$C_s$  AND  $C_t$  ARE COVARIANCE FUNCTIONS AND  $\gamma_s$  AND  $\gamma_t$  ARE THE CORRESPONDENT VARIOGRAM FUNCTIONS.

$C_{st}(0)$  IS THE  $\gamma_{st}$  SILL, BEING  $C_s(0)$  AND  $C_t(0)$  THE  $\gamma_s$  AND  $\gamma_t$  SILLS.

BY DEFINITION,  $\gamma_{st}(0,0) = \gamma_s(0) = \gamma_t(0) = 0$ .



Example of product-sum model, marginal (omnidirectional) spatial and temporal semivariograms (left) and spatiotemporal semivariogram (right).

SOURCE: Denham (2012)

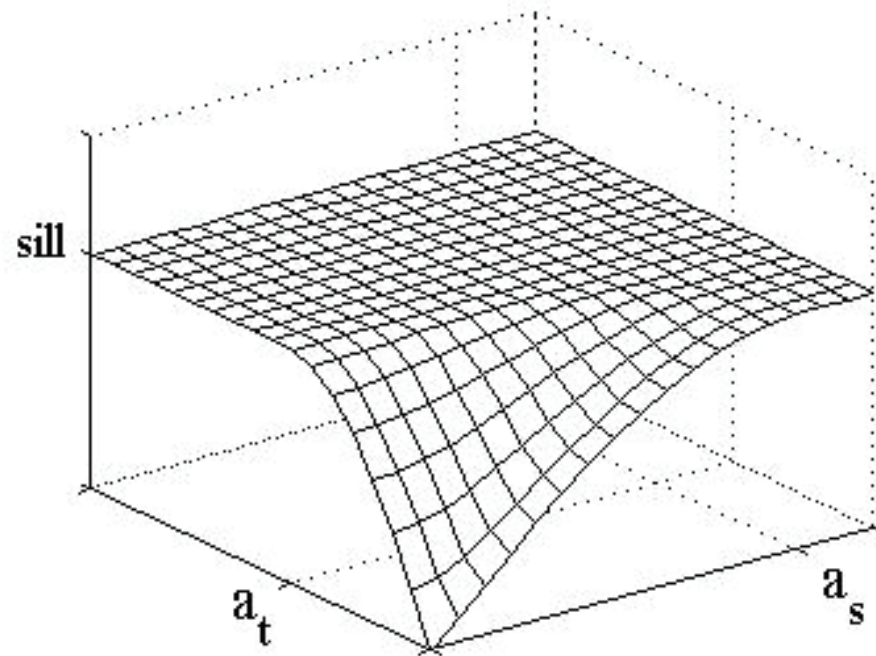
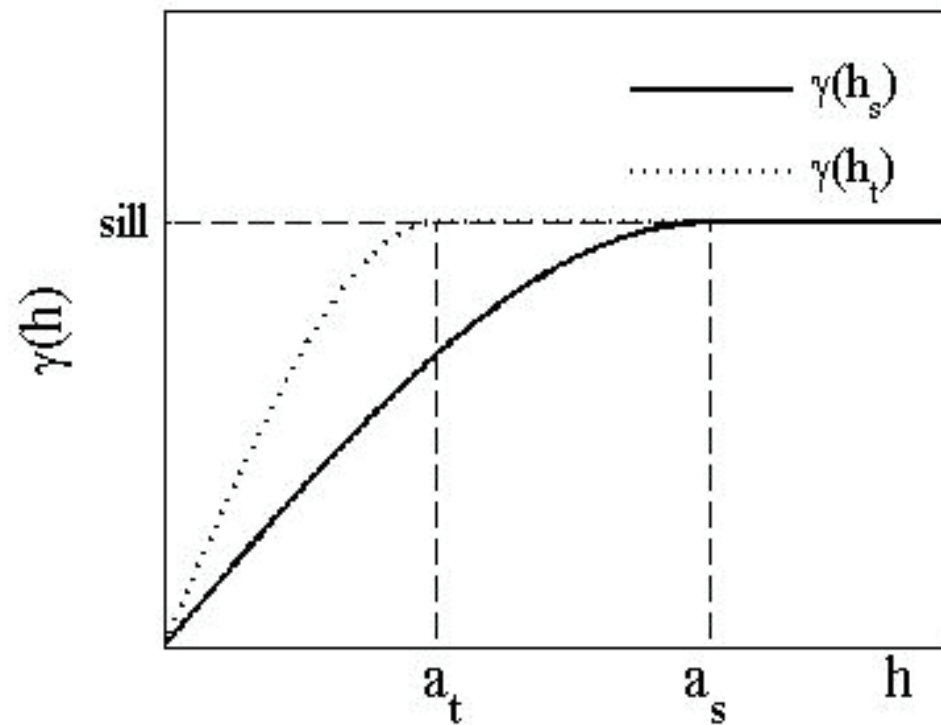
# METRIC

$$C_{st}(h_s, h_t) = C(a^2|h_s|^2 + b^2h_t^2)$$

COEFFICIENTS  $A; B \in R$  .

IN THE EQUATION, THE SAME MODEL IS ASSUMED FOR BOTH SPATIAL AND TEMPORAL COVARIANCE, WITH POSSIBLE CHANGES IN THE RANGE.

THIS MODEL CAN BE REALISED AS A SPATIAL COVARIANCE MODEL WITH AN EXTRA DIMENSION (AND ANISOTROPY RATIO) TO CONSIDER THE TEMPORAL DIMENSION.



Example of metric model, semivariograms in (omnidirectional) spatial and temporal directions (left) and spatiotemporal semivariogram (right).

SOURCE: Denham (2012)



# STUDY AREA AND DATA SET

SANTA BARBARA ECOLOGICAL STATION (EECSB)

MONITORING WATER TABLE DEPTHS FROM SEPTEMBER 2014 TO AUGUST 2016

BAURU AQUIFER SYSTEM

65 WELLS

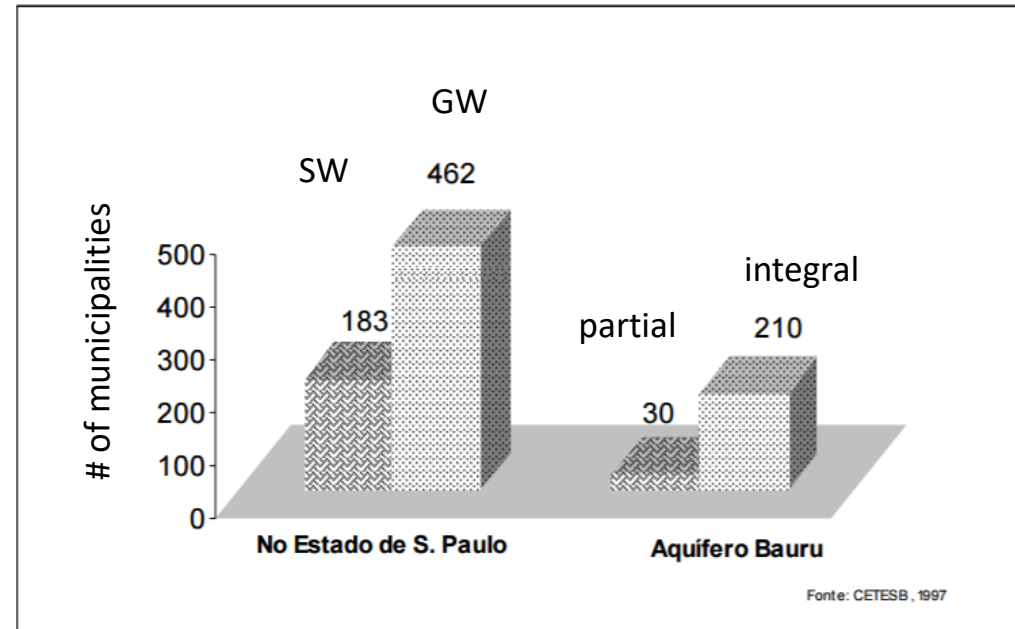
*GSTAT* ROUTINES IN R (PEBESMA & GRÄLER, 2016; GRÄLER ET AL., 2016)



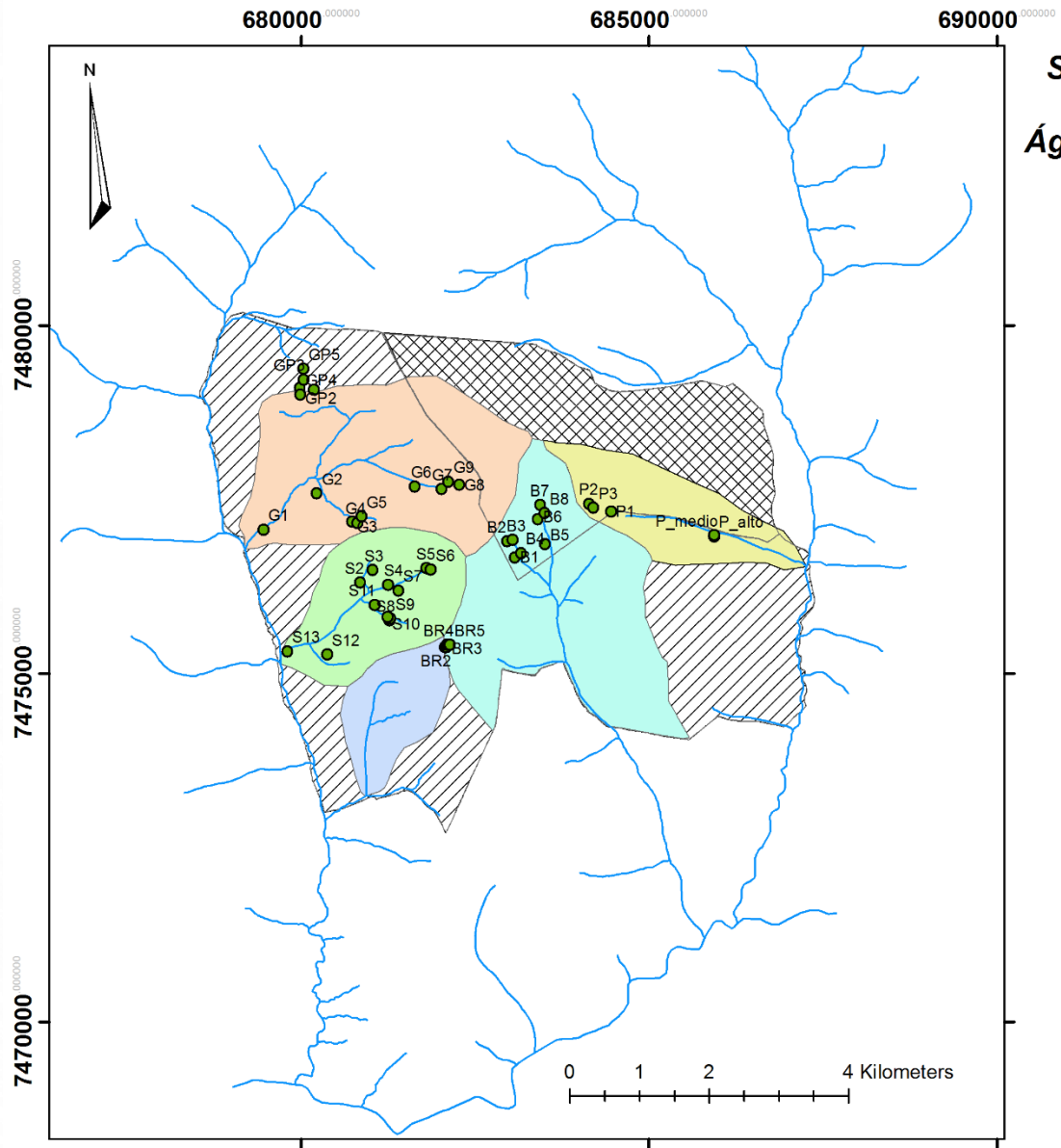




# BAURU AQUIFER SYSTEM (BAS)



COMPARISON AMONG THE NUMBER OF CITIES THAT USE SUPERFICIAL AND UNDERGROUND WATER FOR PUBLIC PROVISIONING IN THE SÃO PAULO STATE AND AMONG THE CITIES THAT USE, PARTIAL OR INTEGRALLY, UNDERGROUND WATER OF THE BAURU AQUIFER SYSTEM (CETESB, 1997).



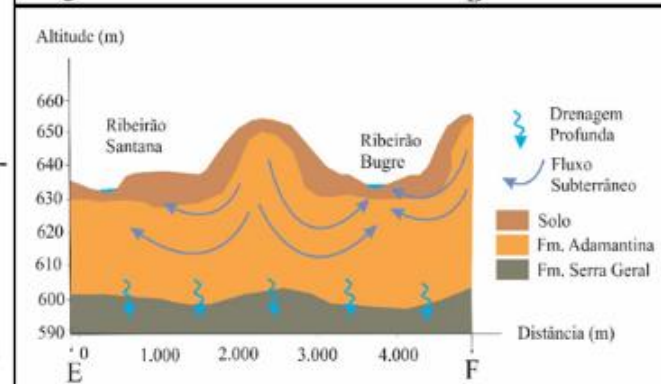
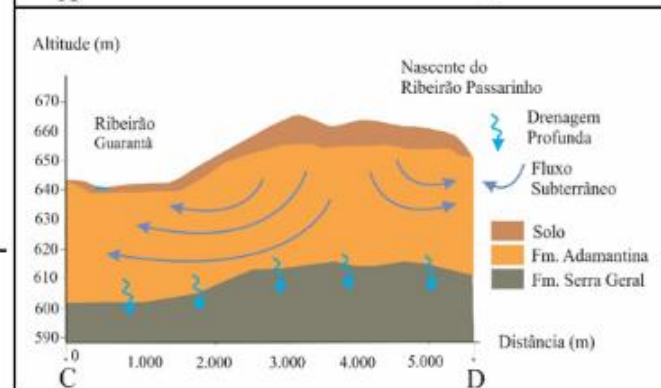
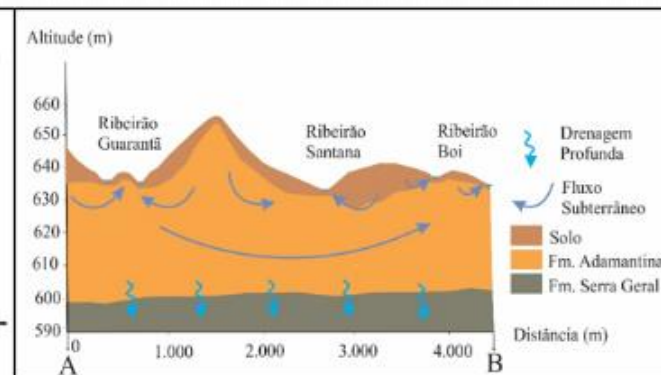
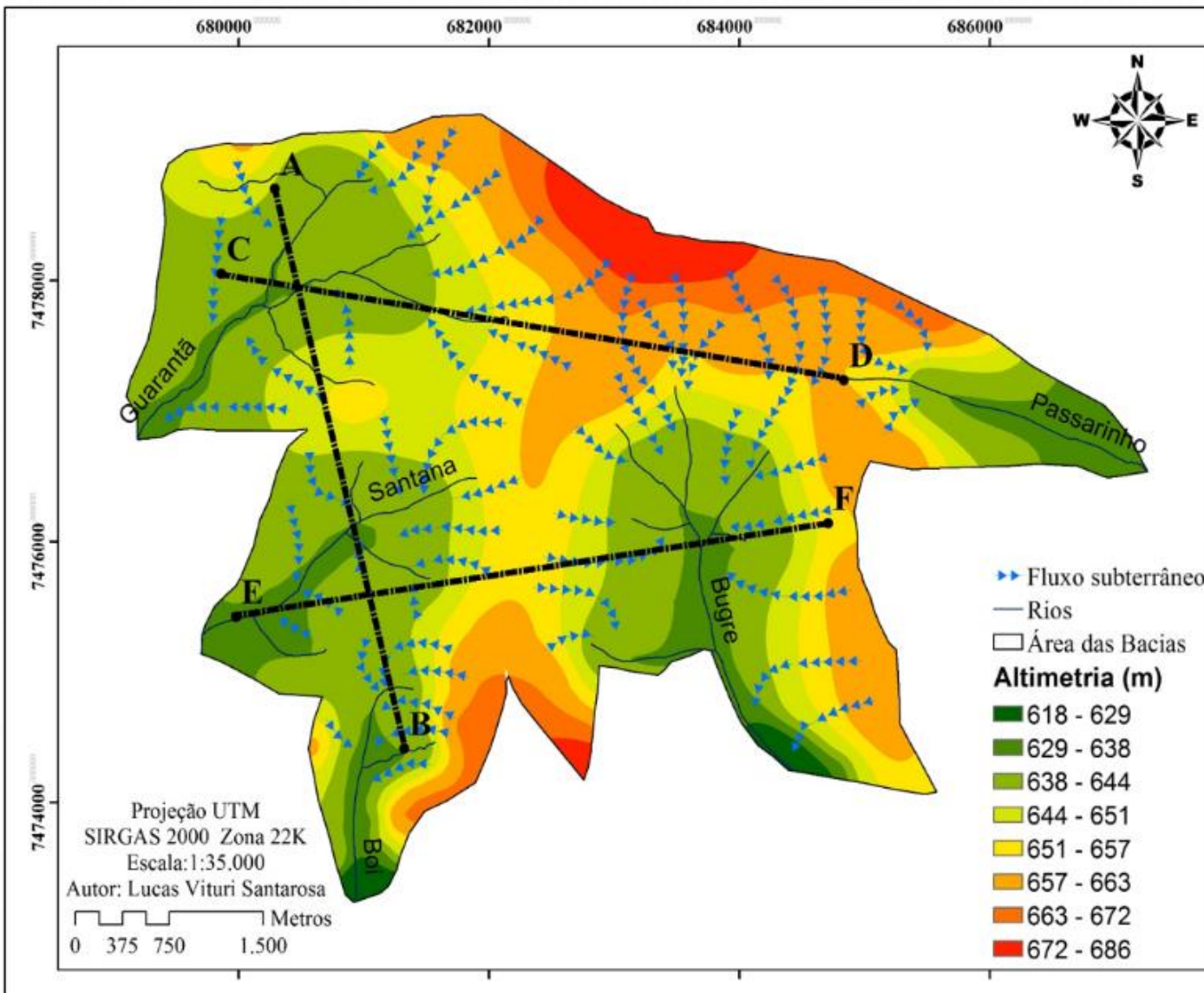
**Santa Bárbara Ecological Station  
(EEcSB)  
Águas de Santa Bárbara - SP, Brazil**

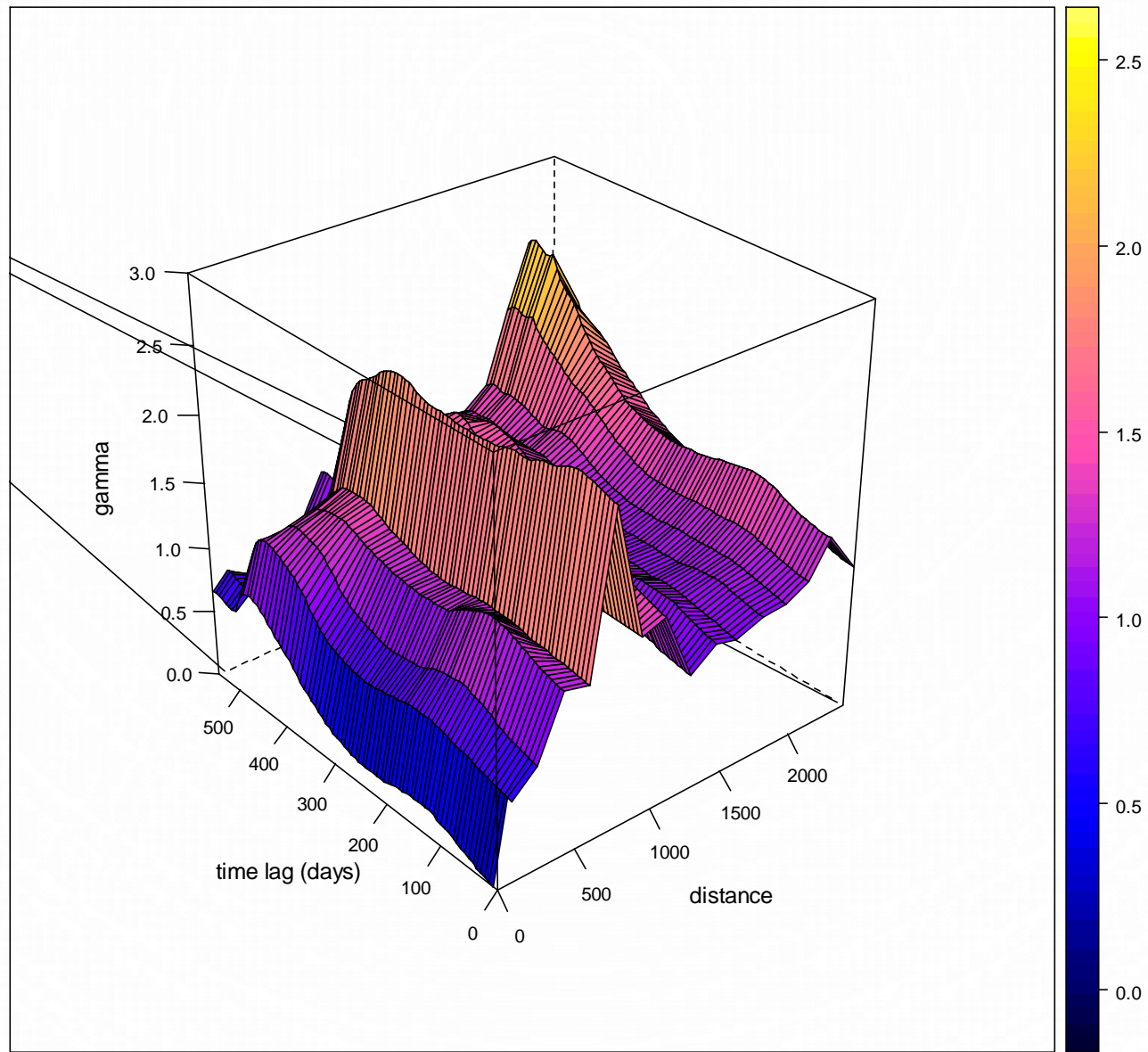
- Wells
  - Drainage
  - ▨ EEcSB
  - ▩ SB State Forrest
- Watersheds**
- Boi
  - Santana
  - Passarinho
  - Gurarantã
  - Bugre

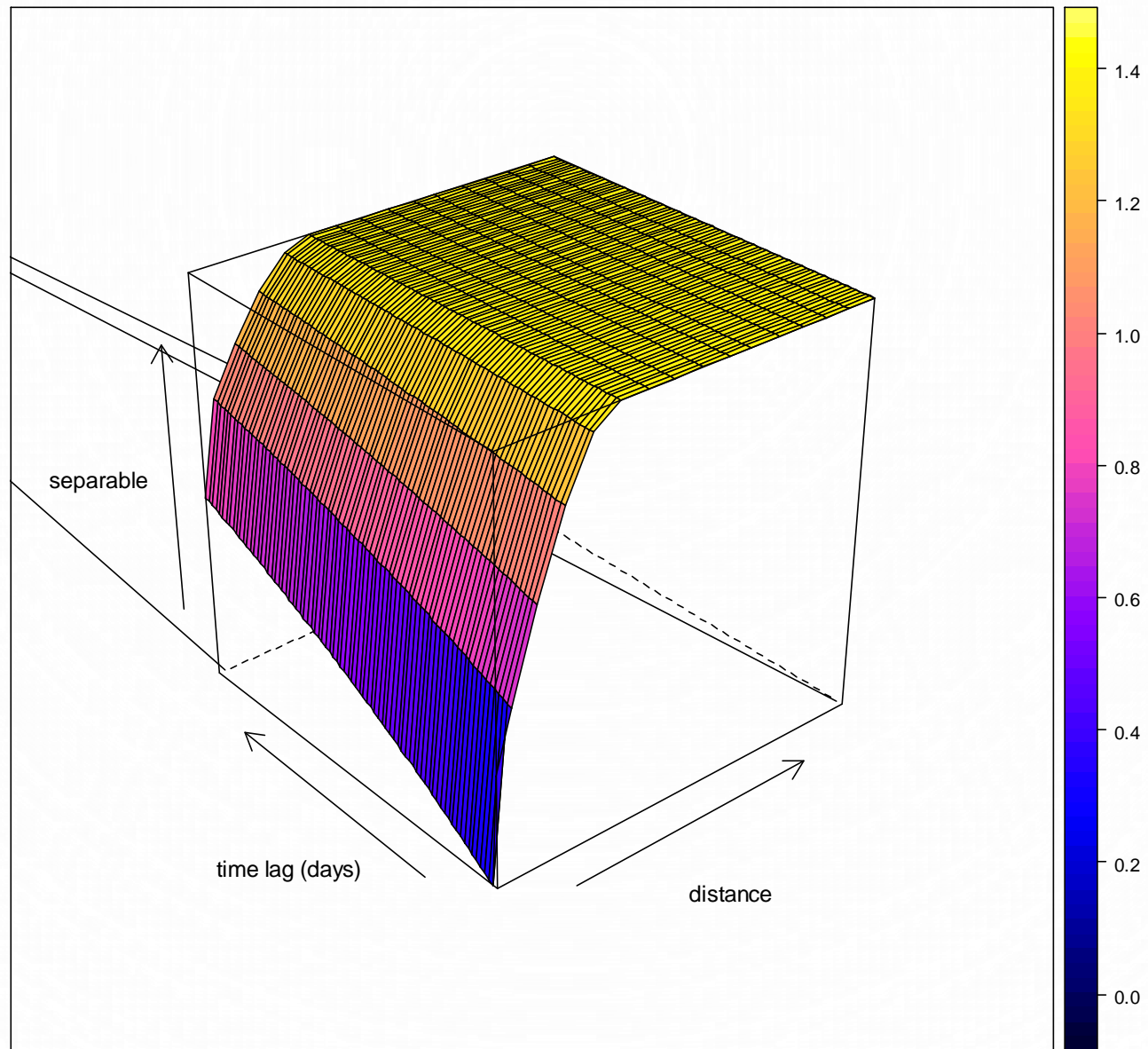
Projection UTM  
Datum: SIRGAS 2000  
Zone 22 k (51°W)

Autor: Prof. Dr. Rodrigo Lilla Manziane  
UNESP/Ourinhos







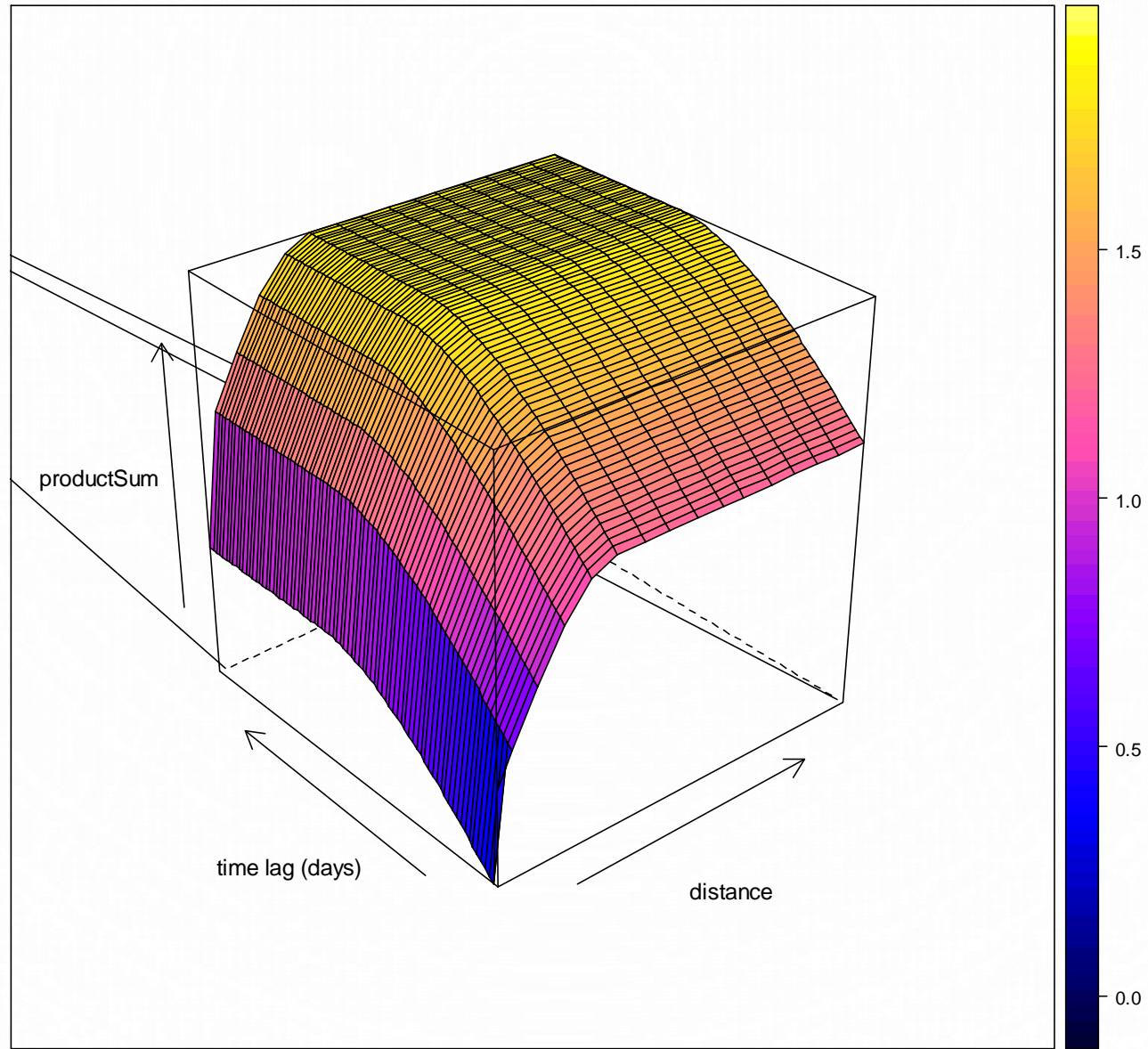


"separable",

space=vgm(0.9,"Sph", 750, 0.1),

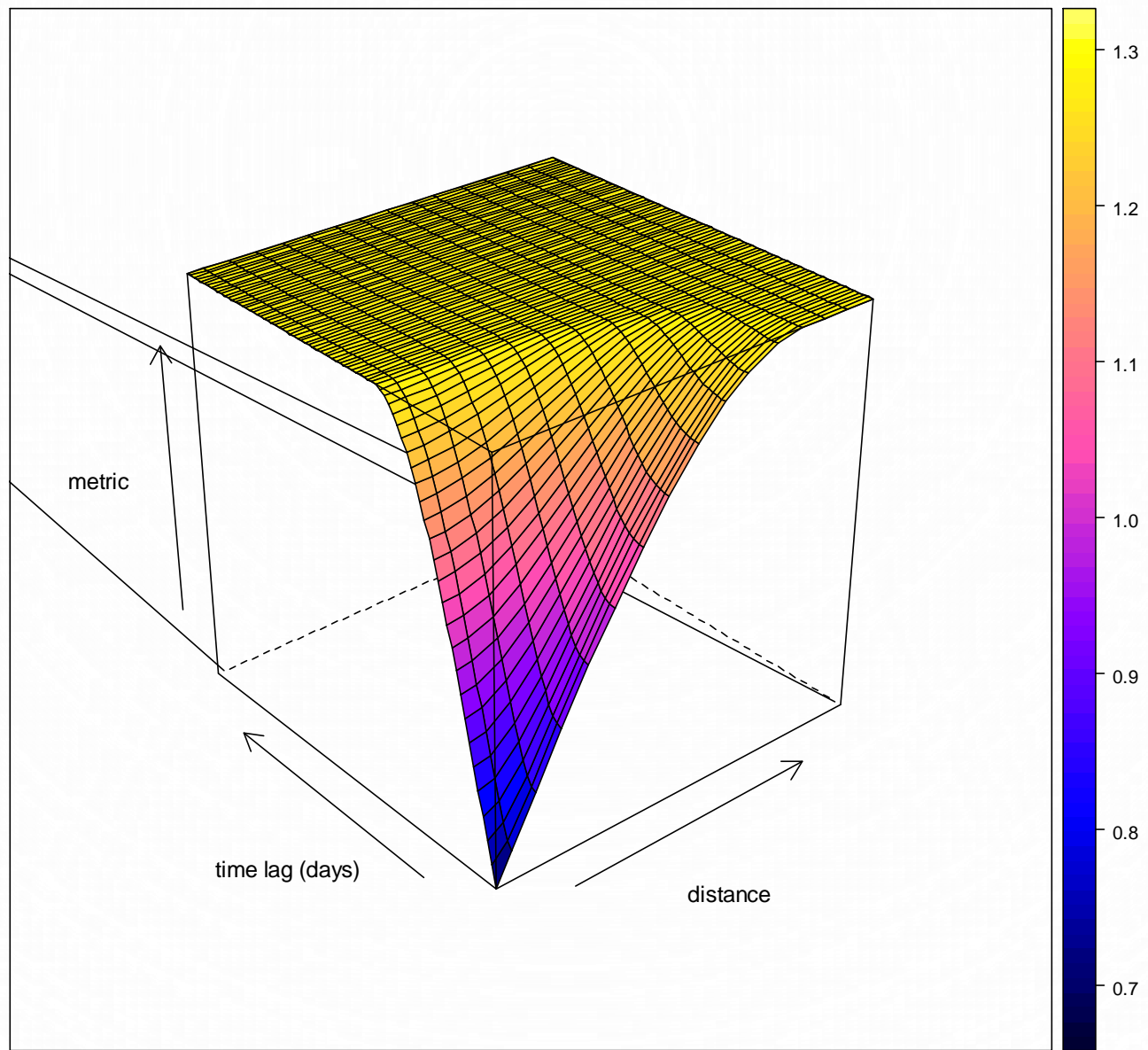
time =vgm(0.8,"Exp", 150, 0.2),

sill=2



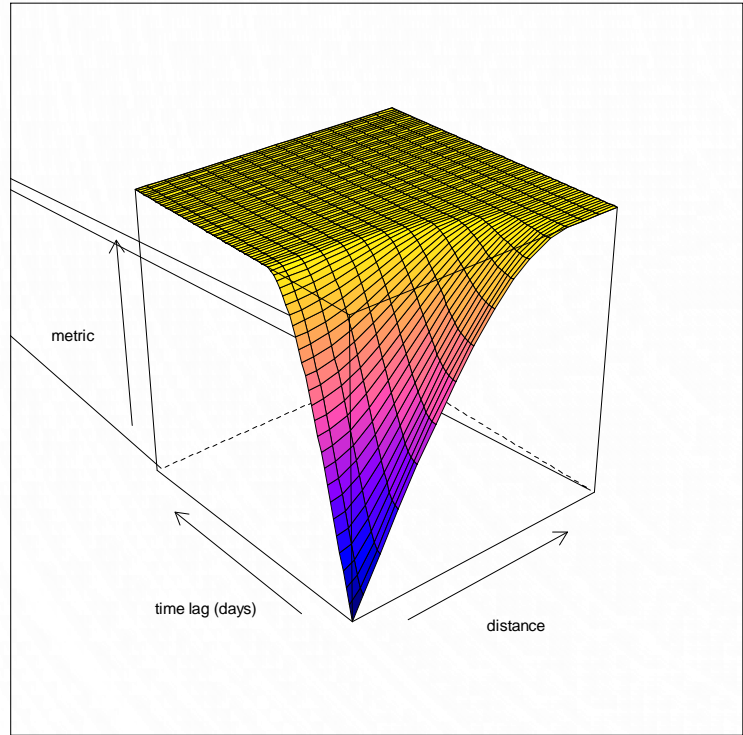
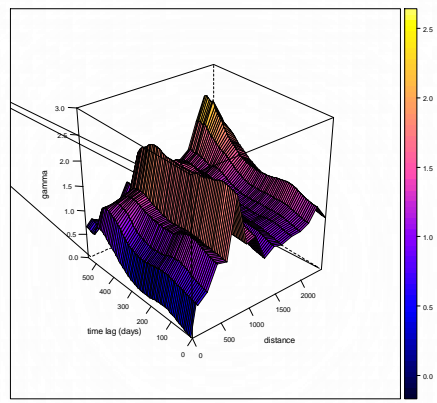
"productSum",

space = vgm(1.2, "Sph", 750, 0.1),  
time = vgm(0.8, "Sph", 320, 0.0),  
k = 2)

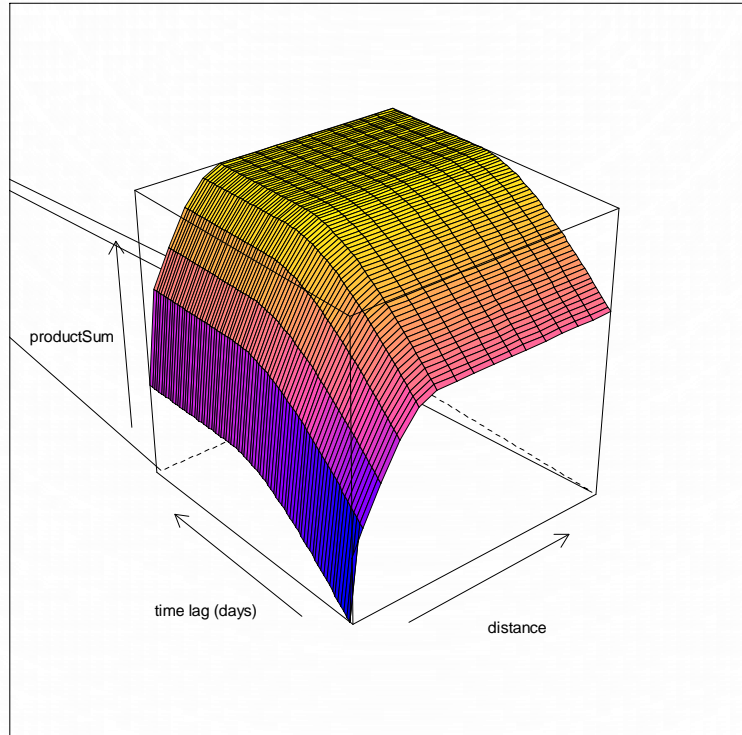


"metric",

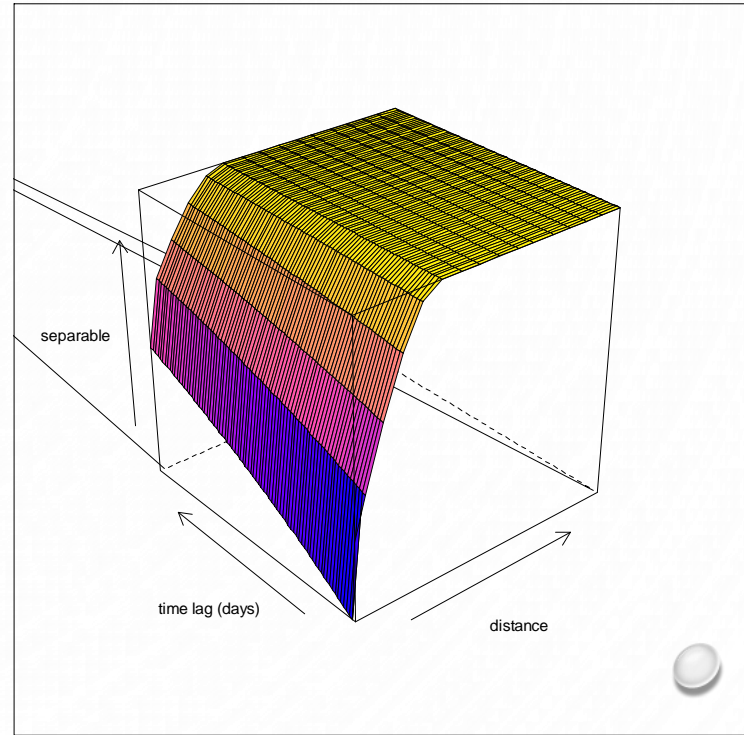
```
joint=vgm(1.7,"Sph", 950, 0.2),  
stAni=10)
```



SEPARABLE




PRODUCT-SUM



METRIC





# FINAL REMARKS – CONCLUSIONS


ST VARIABILITY

NETWORK

PRODUCT-SUM

FAST RESPONSE





# FINAL REMARKS – CONCLUSIONS

GROUNDWATER MANAGEMENT AND PLANNING

MONITORING STRATEGIES

MANAGED AQUIFER RECHARGE

REMEDICATION PLANS





# FINAL REMARKS – NEXT STEPS

INCREASE TS LENGTH

INCREASE OBSERVATION POINTS VIA GEOSTATISTICAL OPTIMIZATION

TEST OTHER MODELS

TEST SEPARABILITY

ST INTERPOLATION OF WATER TABLE DEPTHS



# REFERENCES

- CRESSIE, N.; HUANG, H. C. (1999). CLASSES OF NONSEPARABLE, SPATIO-TEMPORAL STATIONARY COVARIANCE FUNCTIONS. *JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION*, 94, 1330-1340.
- DE CESARE, L.; MYERS, D. E.; POSA, D. (2001) ESTIMATING AND MODELING SPACE-TIME CORRELATION STRUCTURES. *STATISTICS AND PROBABILITY LETTERS*, 51, 9-14.
- DE IACO, S.; MYERS, D. E.; POSA, D. (2001). SPACE-TIME ANALYSIS USING A GENERAL PRODUCT-SUM MODEL. *STATISTICS AND PROBABILITY LETTERS*, 52, 21-28.
- DE IACO, S.; MYERS, D. E.; POSA, D. (2002A) NONSEPARABLE SPACE-TIME COVARIANCE MODELS: SOME PARAMETRIC FAMILIES. *MATHEMATICAL GEOLOGY*, 34, 23-42.
- DE IACO, S.; MYERS, D. E.; POSA, D. (2002B) SPACE-TIME VARIOGRAMS AND A FUNCTIONAL FORM FOR TOTAL AIR POLLUTION MEASUREMENTS. *COMPUTATIONAL STATISTICS AND DATA ANALYSIS*, 41, 311-328.
- DE IACO, S. (2010) SPACE-TIME CORRELATION ANALYSIS: A COMPARATIVE STUDY. *JOURNAL OF APPLIED STATISTICS*, 37, 1027-1041.
- DENHAM, A. (2012) GEOSTATISTICAL SPATIOTEMPORAL MODELLING WITH APPLICATION TO THE WESTERN KING PRAWN OF THE SHARK BAY MANAGED PRAWN FISHERY. PHD THESIS (EDITH COWAN UNIVERSITY)
- DIMITRAKOPOULOS, R.; LUO, X. (1994) SPATIOTEMPORAL MODELLING: COVARIANCES AND ORDINARY KRIGING SYSTEMS *GEOSTATISTICS FOR THE NEXT CENTURY* (PP. 88-93).
- GNEITING, T. (2002) NONSEPARABLE, STATIONARY COVARIANCE FUNCTIONS FOR SPACE-TIME DATA. *JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION*, 97, 590-600.
- GRÄLER, B.; PEBESMA, E. J.; HEUVELINK, G. M. B. *SPATIO-TEMPORAL GEOSTATISTICS USING GSTAT*. IFGI: INSTITUTE FOR GEOINFORMATICS UNIVERSITY OF MÜNSTER. AVAIALE AT: <https://cran.r-project.org/web/packages/gstat/vignettes/spatio-temporal-kriging.pdf>>. ACCESS: SEP. 08, 2016.

# REFERENCES

- KOLOVOS, A.; CHRISTAKOS, G.; HRISTOPULOS, D. T.; SERRE, M. L. (2004) METHODS FOR GENERATING NON-SEPARABLE SPATIOTEMPORAL COVARIANCE MODELS WITH POTENTIAL ENVIRONMENTAL APPLICATIONS. *ADVANCES IN WATER RESOURCES*, 27, 815-830.
- KYRIAKIDIS, P. C.; JOURNEL, A. G. (1999) GEOSTATISTICAL SPACE-TIME MODELS: A REVIEW. *MATHEMATICAL GEOLOGY*, 31(6), 651-684.
- MYERS, D. E.; JOURNEL, A. (1990) VARIOGRAMS WITH ZONAL ANISOTROPIES AND NONINVERTIBLE KRIGING SYSTEMS. *MATHEMATICAL GEOLOGY*, 22, 779-785.
- PEBESMA, E. J.; GRÄLER, B. INTRODUCTION TO SPATIO-TEMPORAL VARIOGRAPHY. IFGI: INSTITUTE FOR GEOINFORMATICS UNIVERSITY OF MÜNSTER. AVAIBLE AT: <[HTTPS://CRAN.R-PROJECT.ORG/WEB/PACKAGES/GSTAT/VIGNETTES/ST.PDF](https://cran.r-project.org/web/packages/gstat/vignettes/st.pdf)>. ACCESS: MAY 08, 2016.
- PORCU, E.; MATEU, J.; SAURA, F. (2008) NEW CLASSES OF COVARIANCE AND SPECTRAL DENSITY FUNCTIONS FOR SPATIO-TEMPORAL MODELLING. *STOCHASTIC ENVIRONMENTAL RESEARCH AND RISK ASSESSMENT*, 22(SUPPLEMENT 1), 65-79.
- RODRIGUES, A.; DIGGLE, P. J. (2010) A CLASS OF CONVOLUTION-BASED MODELS FOR SPATIO-TEMPORAL PROCESSES WITH NON-SEPARABLE COVARIANCE STRUCTURE. *SCANDINAVIAN JOURNAL OF STATISTICS*, 37, 553-567.
- ROUHANI, S.; HALL, T. J. (1989) SPACE-TIME KRIGING OF GROUNDWATER DATA. IN M. ARMSTRONG (ED.), *GEOSTATISTICS: PROCEEDINGS OF THE THIRD INTERNATIONAL GEOSTATISTICS CONGRESS, SEPTEMBER 5-9, 1988, AVIGNON, FRANCE* (VOL. 2, PP. 639-650). DORDRECHT, BOSTON: KLUWER ACADEMIC PUBLISHERS.
- ROUHANI, S.; MYERS, D. E. (1990) PROBLEMS IN SPACE-TIME KRIGING OF HYDROGEOLOGICAL DATA. *MATHEMATICAL GEOLOGY*, 611-623.
- SOARES, A.; PEREIRA, M. J. (2007) SPACE-TIME MODELLING OF AIR QUALITY FOR ENVIRONMENTAL-RISK MAPS: A CASE STUDY IN SOUTH PORTUGAL. *COMPUTERS & GEOSCIENCES*, 33, 1327-1336.
- STEIN, M. L. (2005) SPACE-TIME COVARIANCE FUNCTIONS. *JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION*, 100, 310-321.

*THANK YOU FOR YOUR ATTENTION*

[RLMANZIONE@GMAIL.COM](mailto:RLMANZIONE@GMAIL.COM)



PÓS-GRADUAÇÃO  
**IRRIGAÇÃO  
E DRENAGEM**  
UNESP - FCA

