Kansa's Multiquadric Based Meshfree Solution for Confined Aquifer
(nº 1534)

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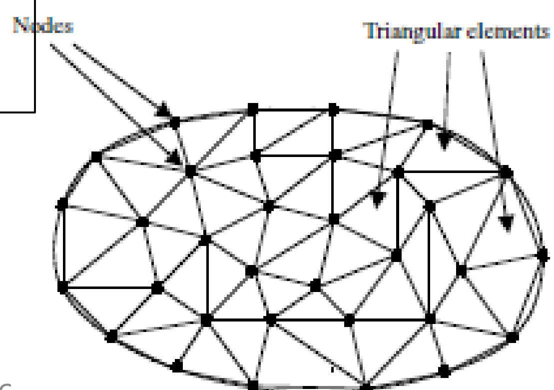
N°abstract

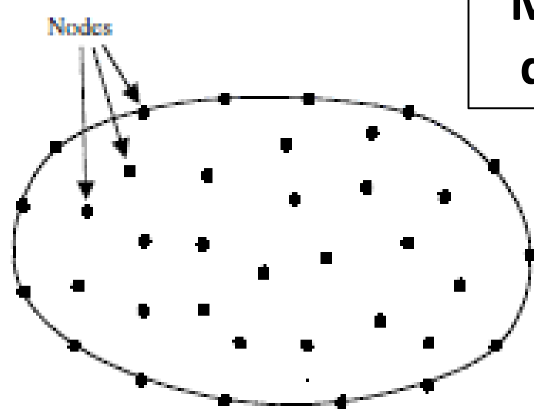
Meshfree (Mfree) methods

- *According to GR Liu (2003)* "An Mfree technique is a method used to establish system algebraic equations for the whole problem domain without the use of predefined mesh for domain discretization".

FEM domain discretization



Mfree domain discretization



Governing equation for confined aquifer

- Confined anisotropic heterogeneous and areal recharge including pumping (Willis and Yeh 1987):

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right)=S\frac{\partial h}{\partial t}\pm Q_w\left(x-x_p\right)\left(y-y_p\right)+R$$

- Initial boundary condition: $h(x,y,0)=h_0(x,y)$
- Constant head: $h(x,y,t)=h_1(x,y,t)$
- Boundary flux: $T\dfrac{\partial h}{\partial n}=q_2(x,y,t)$

Approximation of head variable

- Head approximation: If $h(x,y,t) \rightarrow h(x,y,t)$
- Than by multiquadric approach (Kansa 1990):

$$h(x,y,t) = \sum_{j=1}^{N} h_j(t).\phi_j(x,y)$$

- where $\phi_j = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + C_s}$ as RBF

$C_s =$ Shape parameter $= d_s \alpha_s$

$\alpha_s =$ Support size for RBF

$d_s =$ Avg. nodal spacing $= \dfrac{\sqrt{A}}{(\sqrt{N} - 1)}$

A= Area of domain

N= Total no. of nodes in domain

Discretized form of GW flow eq.

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$$\underbrace{\left[\frac{S}{\Delta t}\left(\sum_{j=1}^{N}\phi_j(x_i,y_i)\right)-T_x\left(\sum_{j=1}^{N}\frac{\partial^2\phi_j(x_i,y_i)}{\partial x^2}\right)-T_y\left(\sum_{j=1}^{N}\frac{\partial^2\phi_j(x_i,y_i)}{\partial y^2}\right)\right]}_{A_i}$$

Internal nodes

$$\times\underbrace{\{h_j\}^{t+1}}_{h_j}=\underbrace{\frac{S}{\Delta t}\left(\sum_{j=1}^{N}\phi_j(x_i,y_i).\{h_j\}^{t}\right)}_{f(x_i,y_i)}\ \text{where i=1,2...N}_{\text{I}}$$

Discretized form of GW flow eq.

$$\underbrace{\left[\frac{S}{\Delta t}\left(\sum_{j=1}^{N}\phi_j(x_i,y_i)\right)-T_x\left(\sum_{j=1}^{N}\frac{\partial^2\phi_j(x_i,y_i)}{\partial x^2}\right)-T_y\left(\sum_{j=1}^{N}\frac{\partial^2\phi_j(x_i,y_i)}{\partial y^2}\right)\right]}_{A_I}$$

Internal nodes

$$\times\underbrace{\{h_j\}^{t+1}}_{h_j}=\underbrace{\frac{S}{\Delta t}\left(\sum_{j=1}^{N}\phi_j(x_i,y_i).\{h_j\}^{t}\right)}_{f(x_i,y_i)}$$ where $i=1,2\ldots N_I$

$$\underbrace{\sum_{j=1}^{N}\phi_j(x_i,y_i)}_{A_{BI}}\times\underbrace{\{h_j\}^{t+1}}_{h_j}=\underbrace{h_1(x,y,t)}_{g(x_i,y_i)}$$ where $i=N_I+1\ldots.N_{BI}$

Constant head

$$\underbrace{\left[\left(T_x\sum_{j=1}^{N}\frac{\partial\phi_j(x_i,y_i)}{\partial x}\right)l_x+\left(T_y\sum_{j=1}^{N}\frac{\partial\phi_j(x_i,y_i)}{\partial y}\right)l_y\right]}_{A_{BII}}\times\underbrace{\{h_j\}^{t+1}}_{h_j}=\underbrace{q_2(x,y,t)}_{k(x_i,y_i)}$$

where $i=N_{BI}+1\ldots.N$

Boundary flux

Solution of GW flow eq.

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$$\{h_j\} = [A]^{-1}\{F\}$$

$$\text{hence } h(x,y,t) = [A]^{-1}\{F\}[\phi(x,y,)]$$

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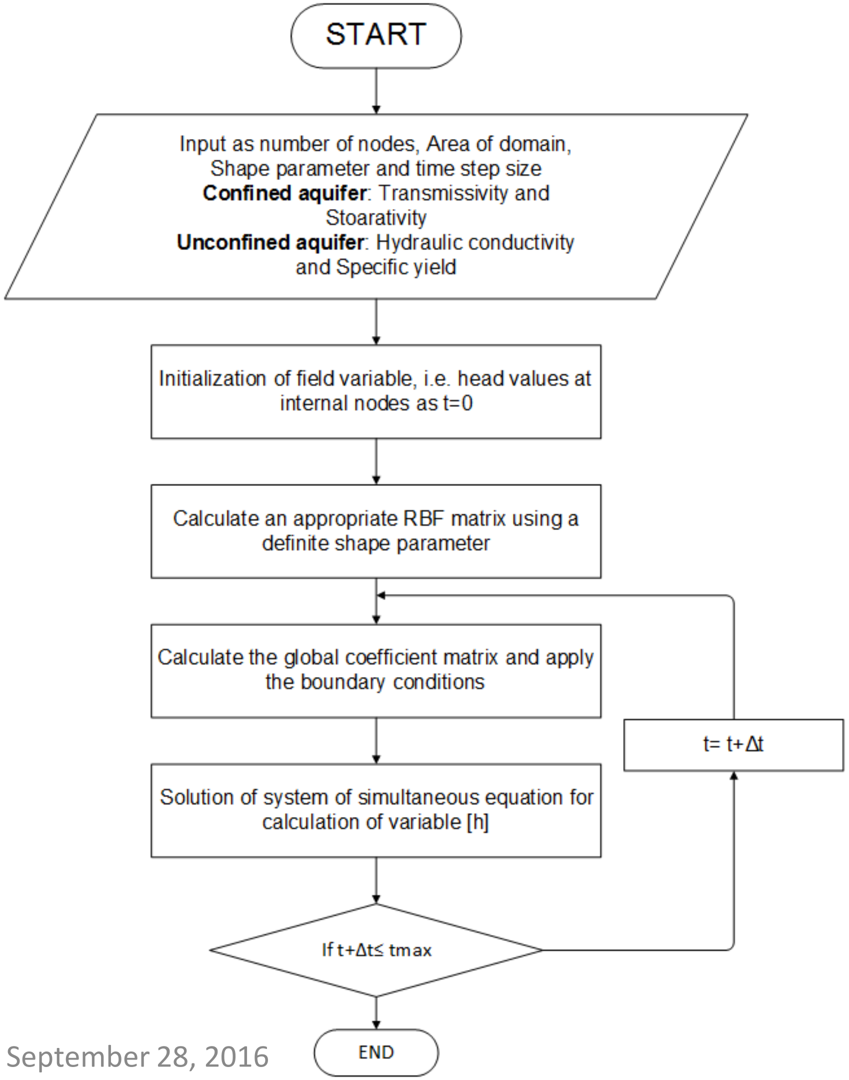
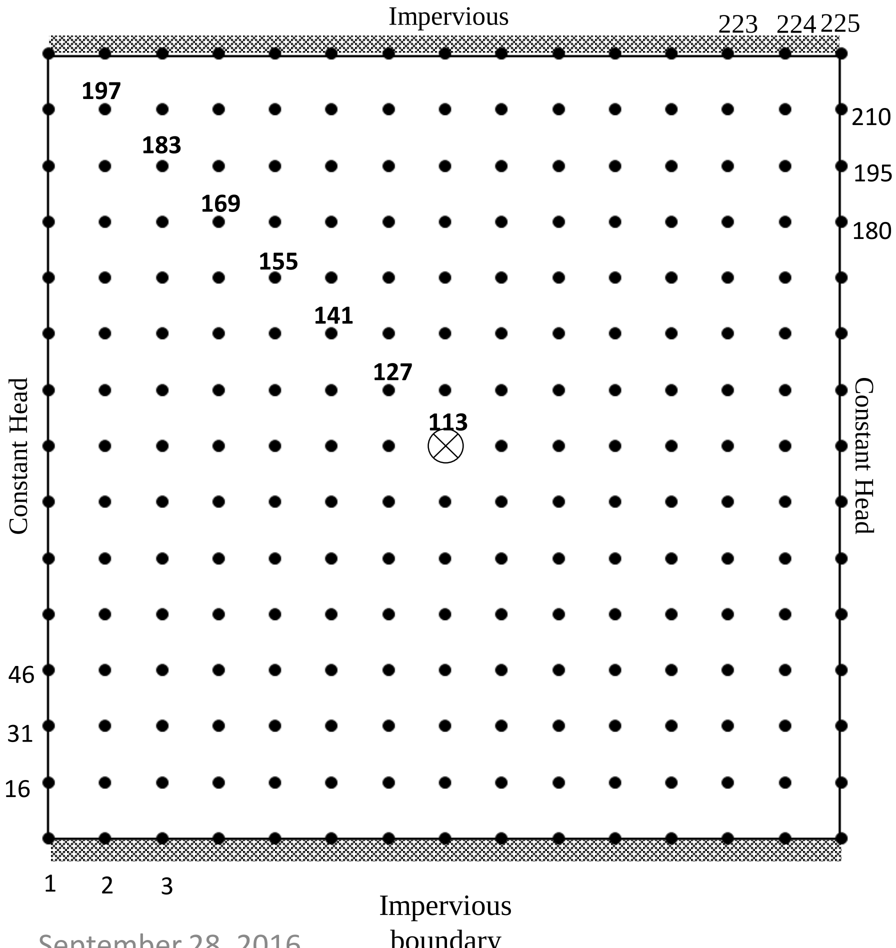
$$\text{where } A = \begin{bmatrix} A_I \\ A_{BI} \\ A_{BII} \end{bmatrix} \text{ and } F = \begin{Bmatrix} f(x_i, y_i) \\ g(x_i, y_i) \\ k(x_i, y_i) \end{Bmatrix}$$

Proposed Meshfree groundwater model



Testing of Mfree simulation model

2-D rectangular well at center problem (Chan et al. 1976)



Testing of Mfree simulation model

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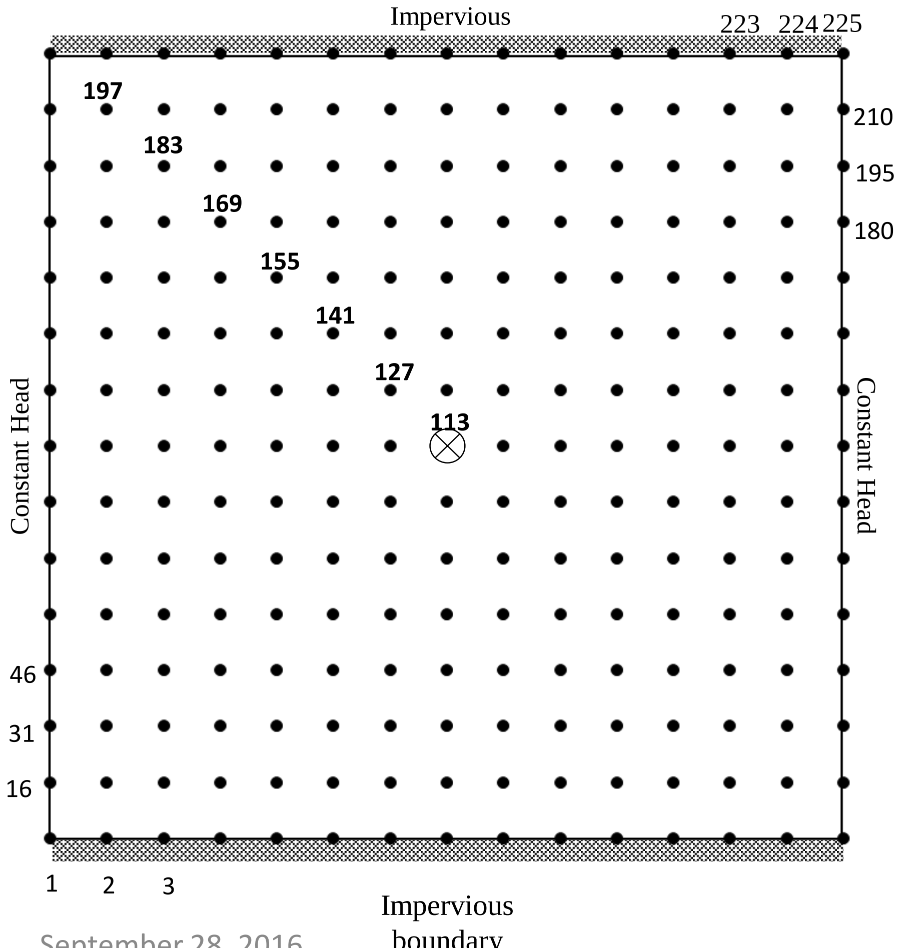
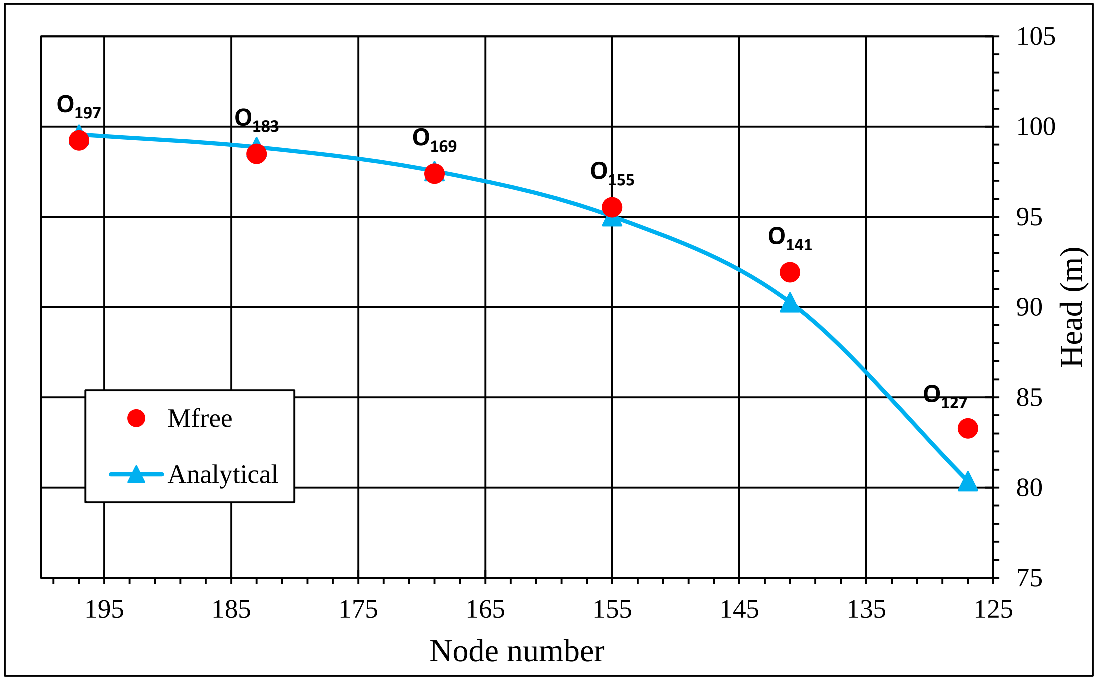


- **Area= 1400m X 1400m**
- **Constant boundary head= 100m**
- **Transmissivity= 100 m^2/d**
- **Initial steady state head= 100 m**
- **Pumping rate at center well = 10000 m^3/d**
- **Number of nodes = 225**

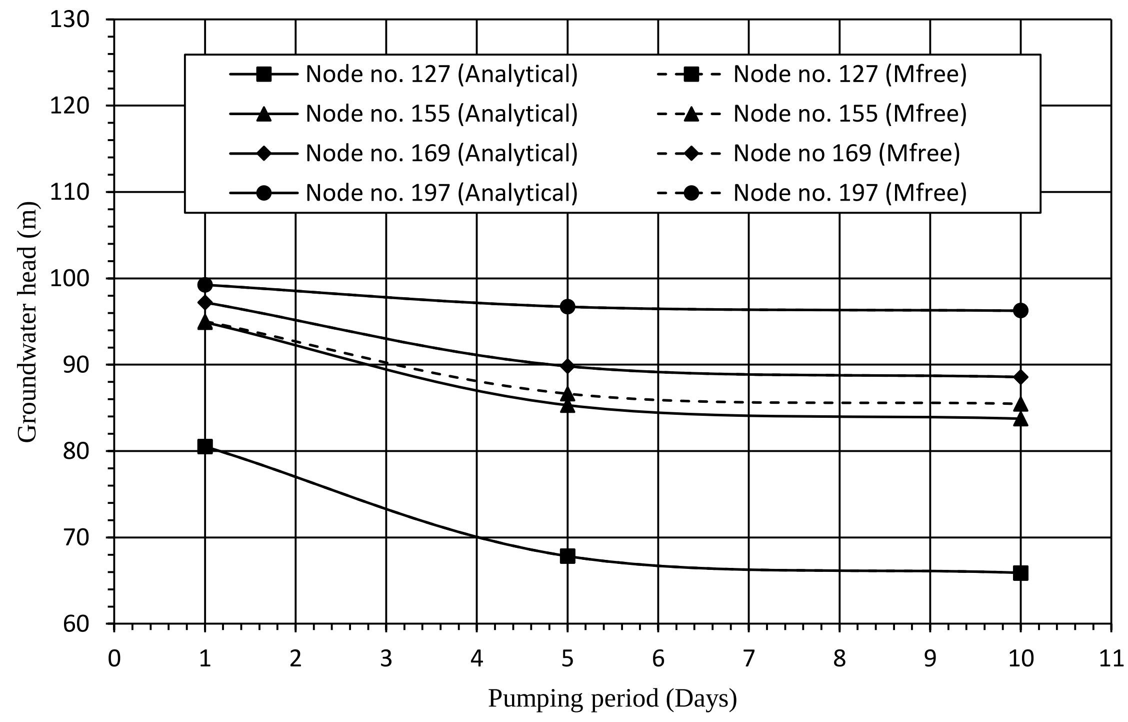
Testing of Mfree simulation model (cont.)

Analytical and Mfree solution (Δt= 1day) for 1 day of pumping

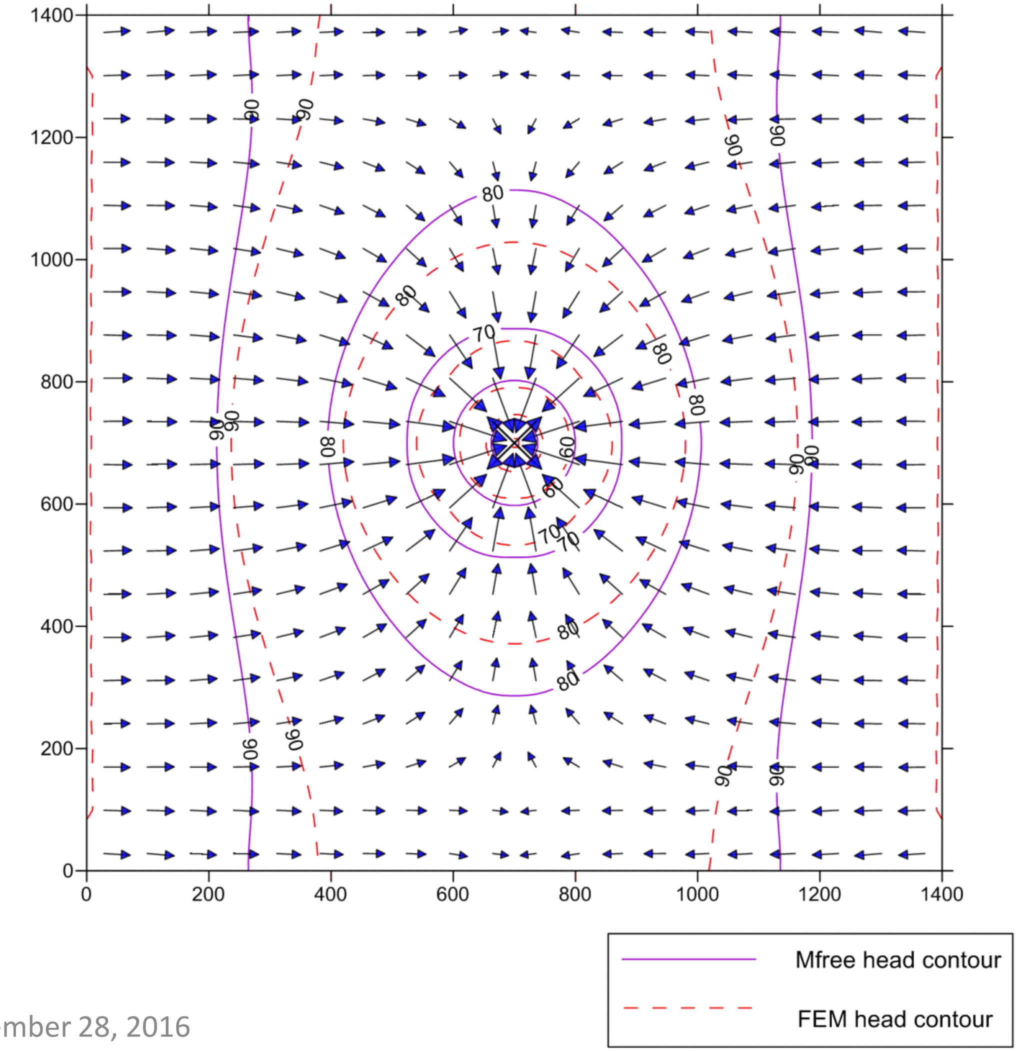


Testing of Mfree simulation model (cont.)

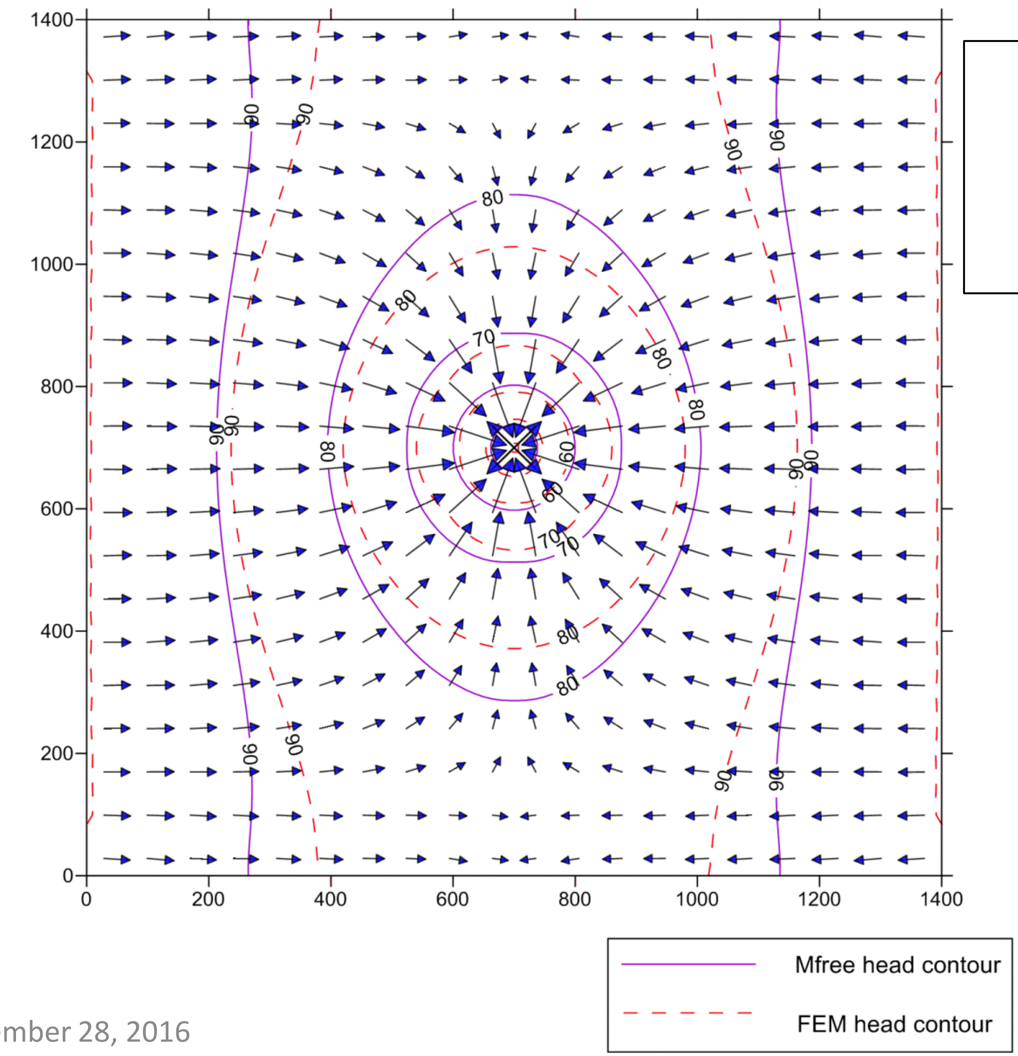
Effect of pumping period on observation well head values



Testing of Mfree simulation model (cont.)



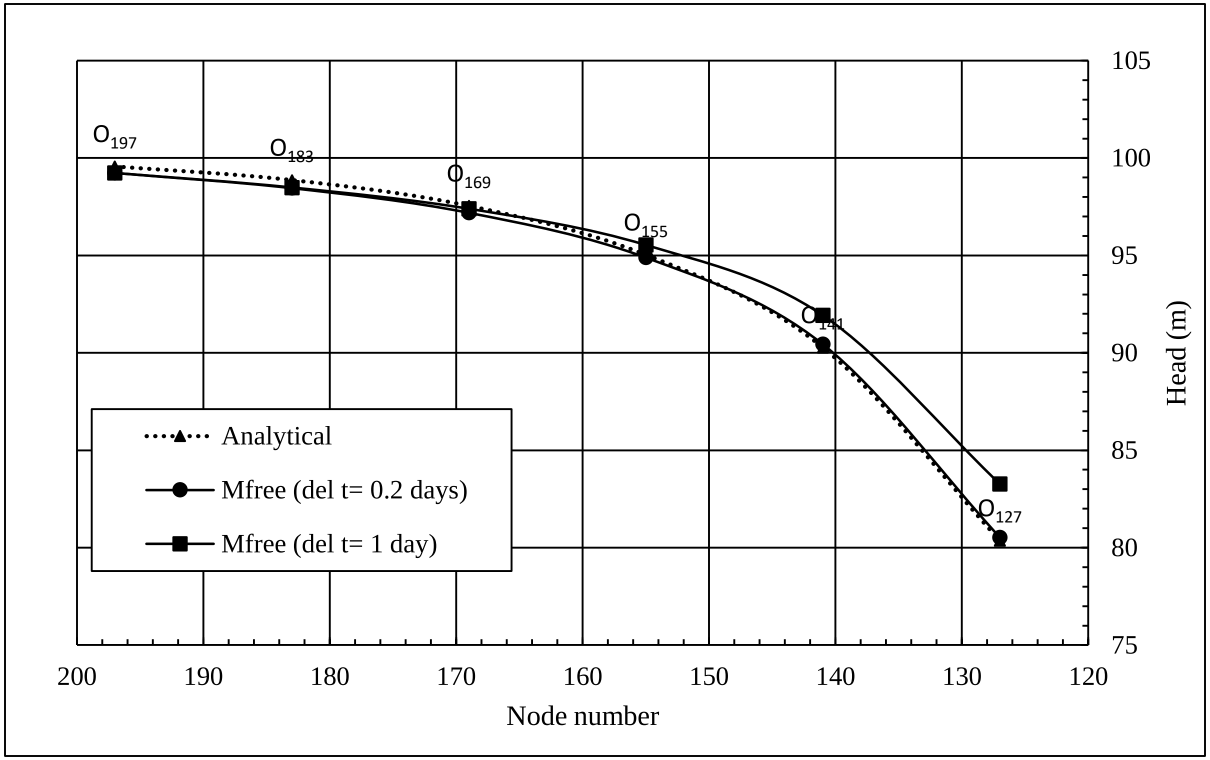
Testing of Mfree simulation model (cont.)



FEM and Mfree groundwater head contours

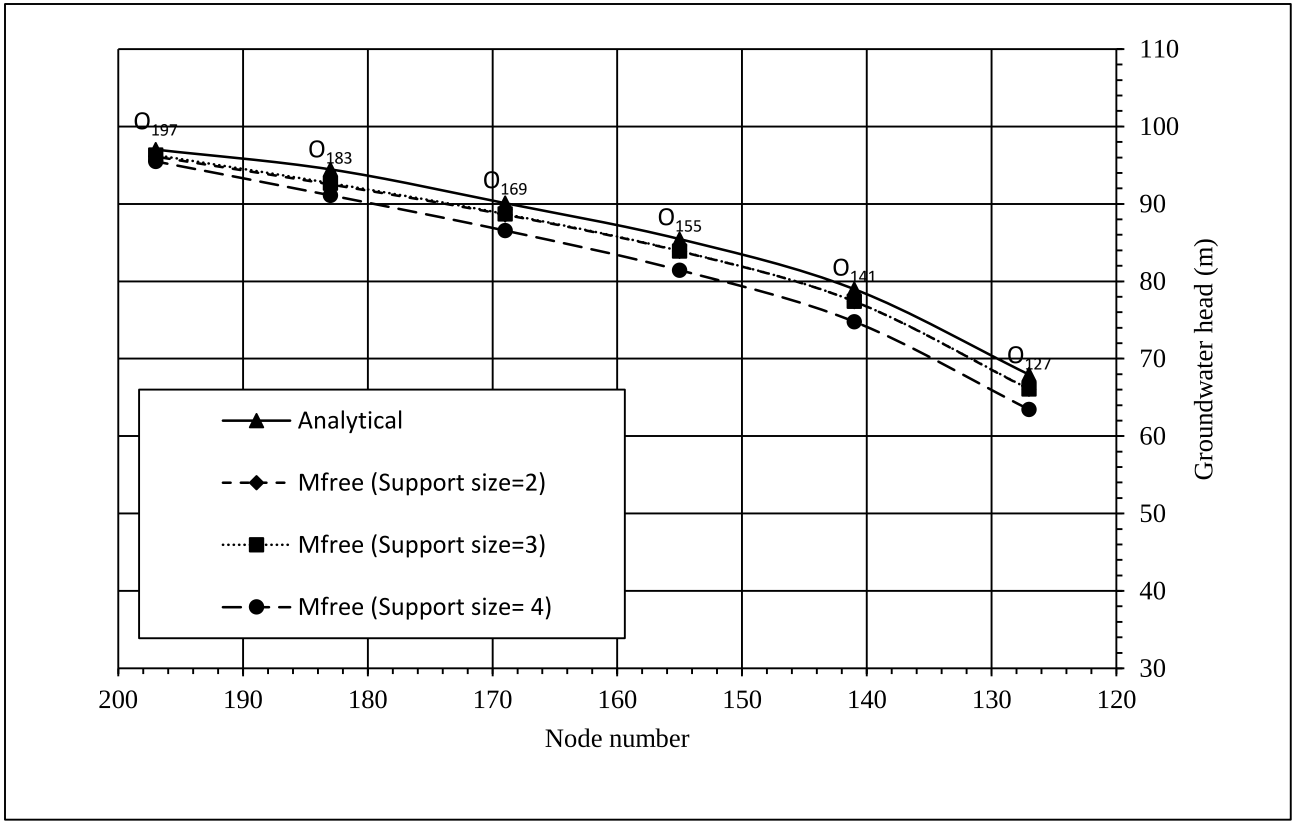
Sensitivity analysis

Effect of time- step size



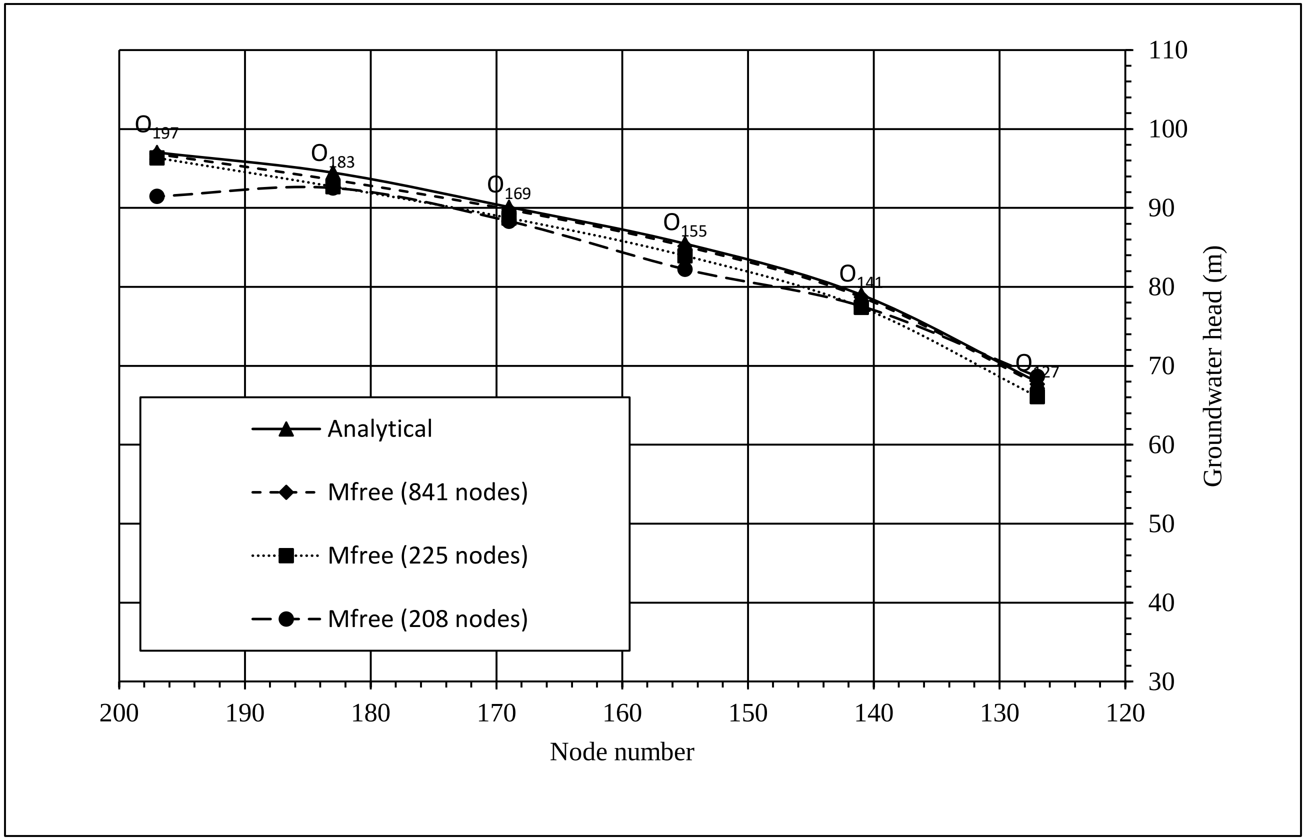
Sensitivity analysis

Effect of support size



Sensitivity analysis

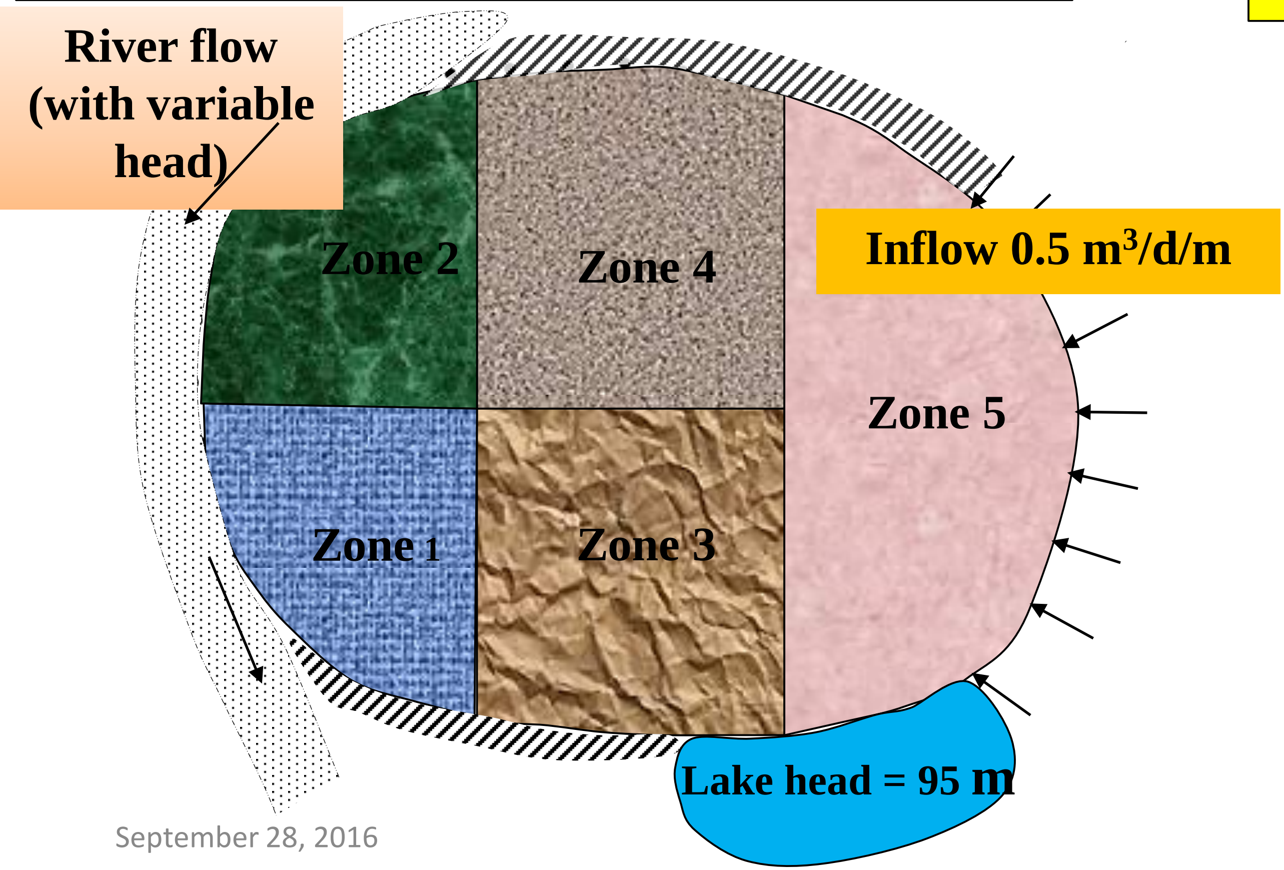
Effect of nodal density



Mfree model application

Irregular heterogeneous synthetic aquifer with flux inflow and temporal river head variation (Cyriac and Rastogi 2016)

Area= 40 km^2
Thickness= 100 m



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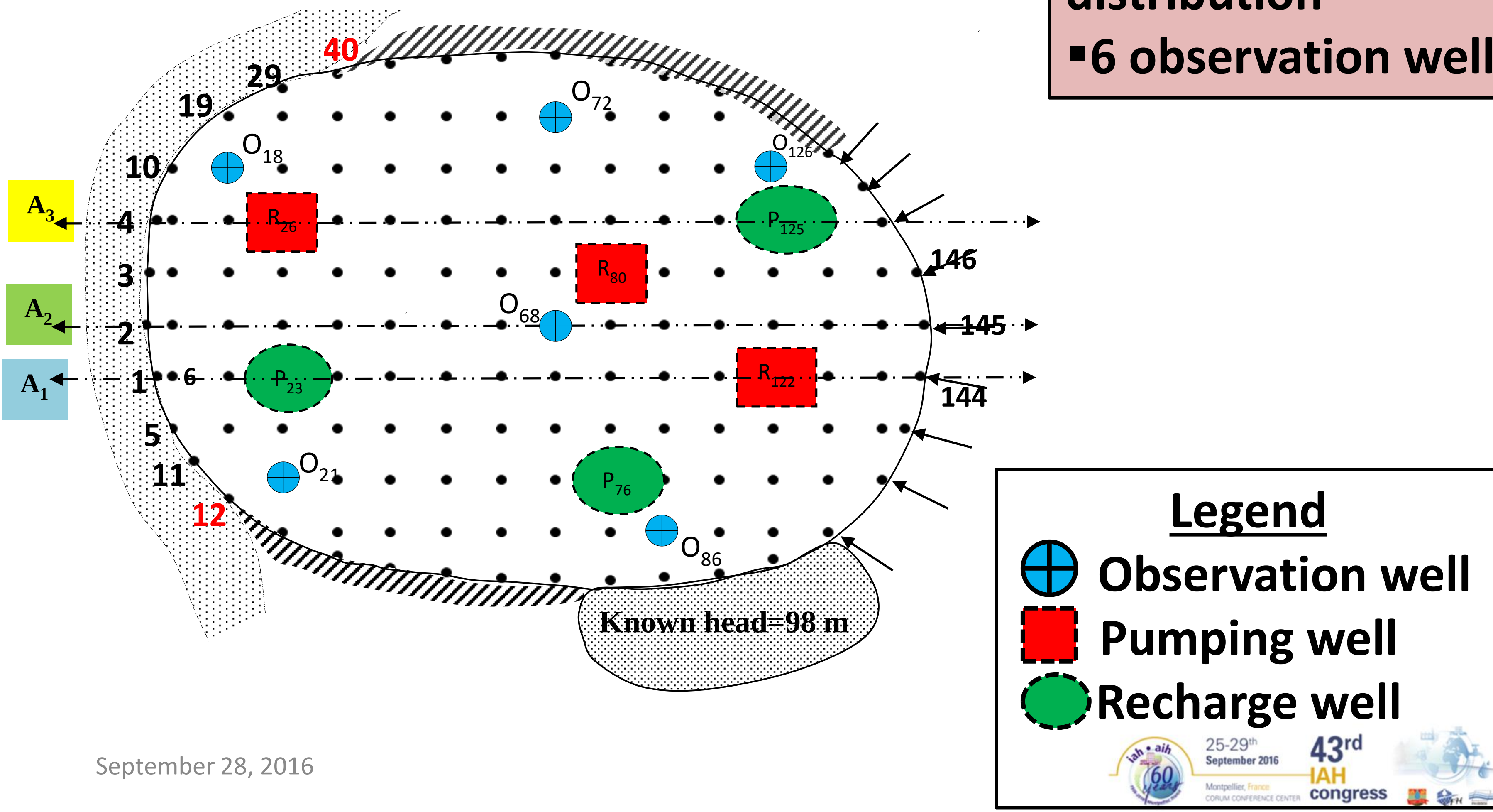
Zones	T_x (m^2/day)	T_y (m^2/day)	Storativity (S)	Zonal area (km^2)
1	1500	1200	0.0004	4.72
2	800	600	0.0003	5.49
3	1000	800	0.0002	7.32
4	1300	1000	0.0001	7.67
5	2000	1000	0.0006	10.49

Mfree model application (cont.)

Nodal distribution with pumping activities

- No of nodes= 146
- Non- uniform nodal distribution
- 6 observation wells



Mfree model application (cont.)

Nodal distribution with pumping activities

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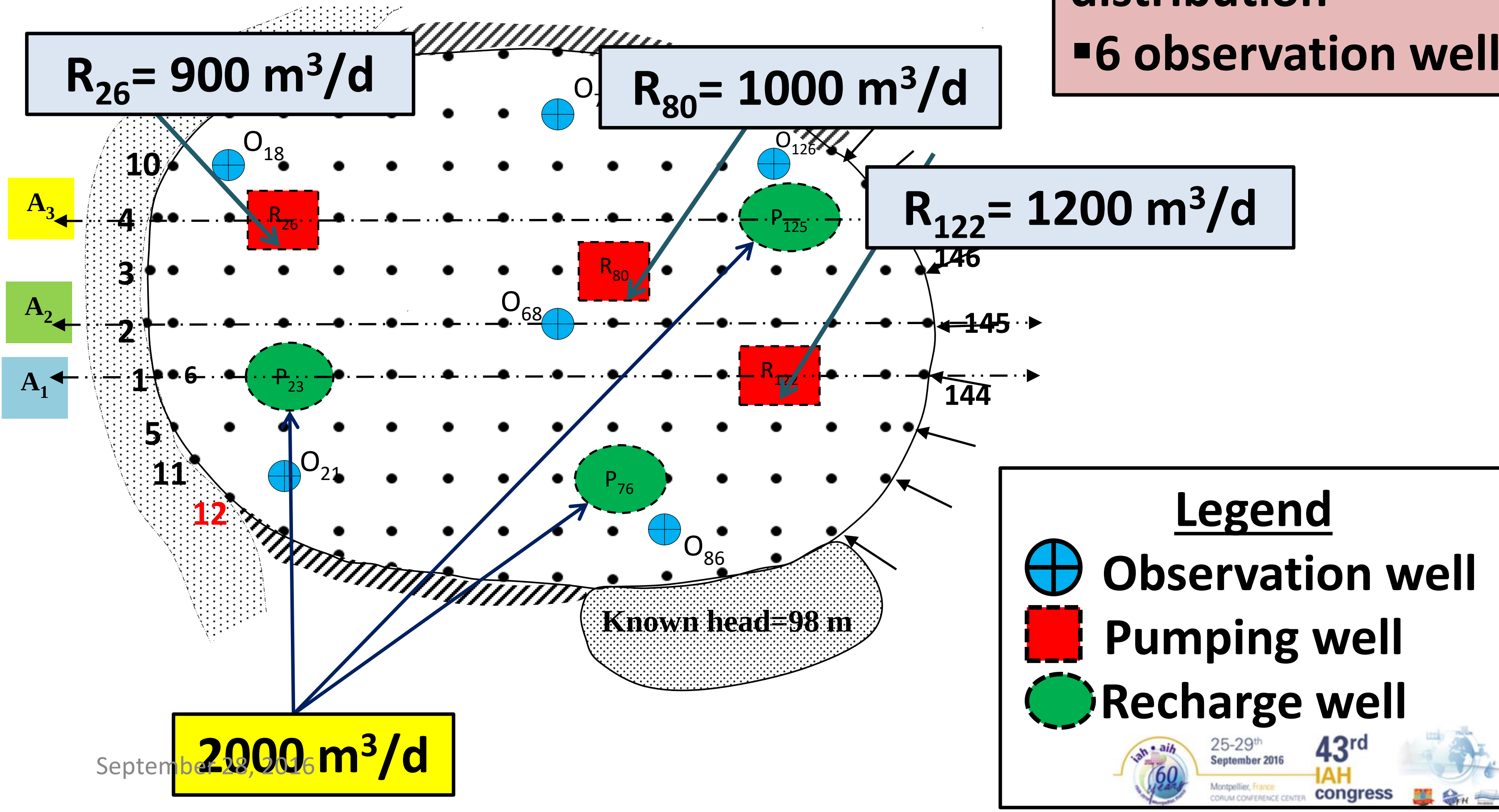
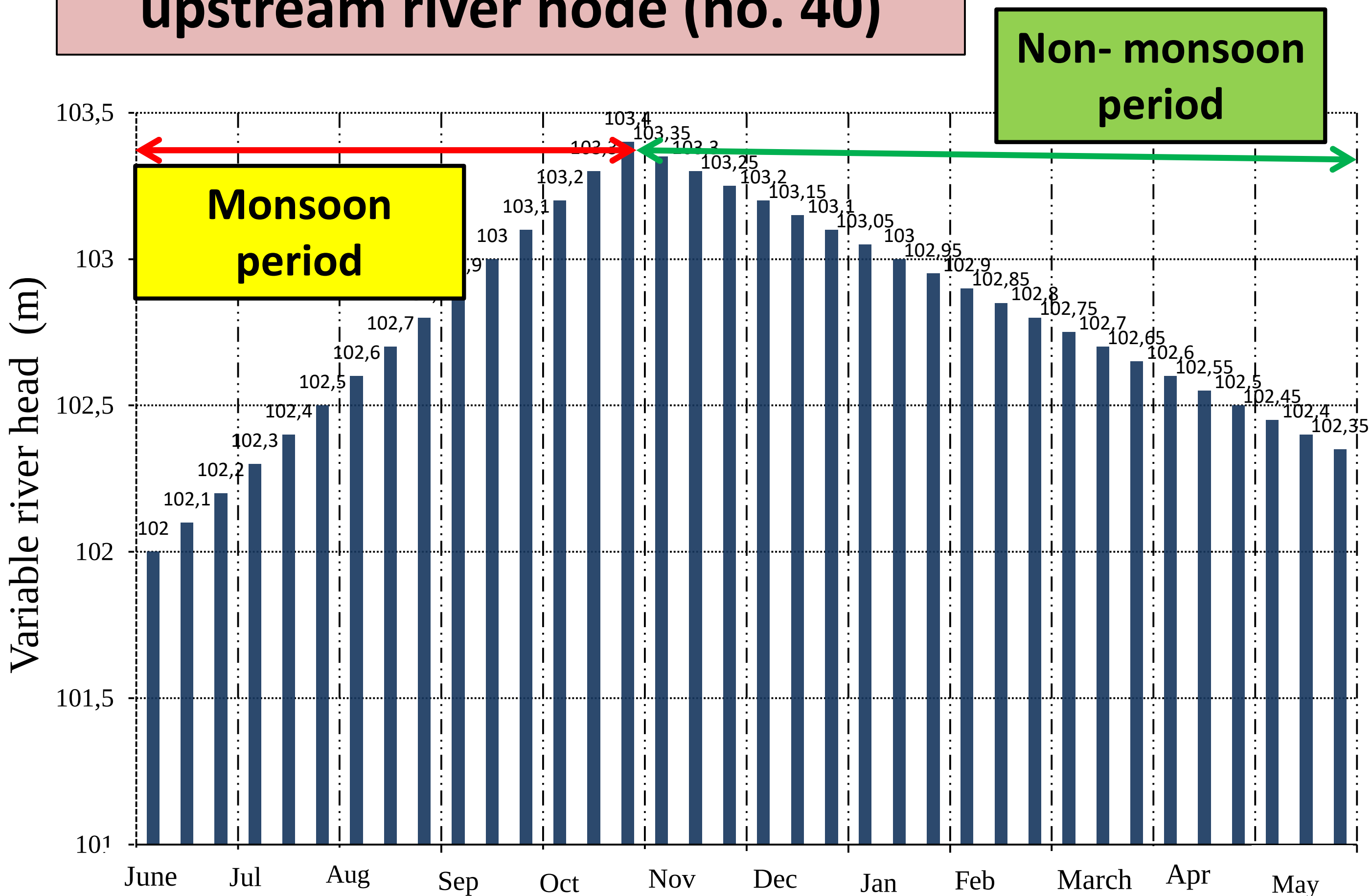


Mfree model application (cont.)

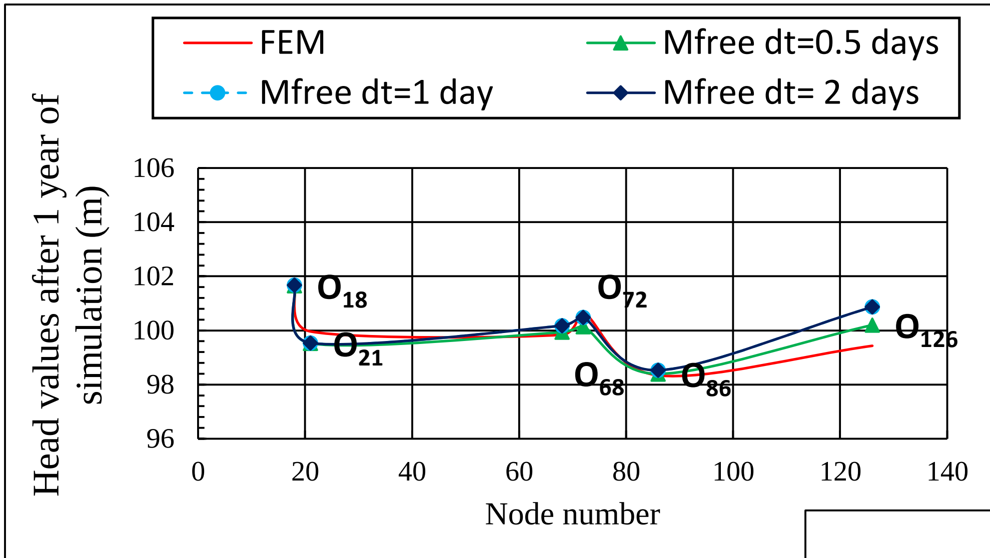
Temporal head variation in upstream river node (no. 40)



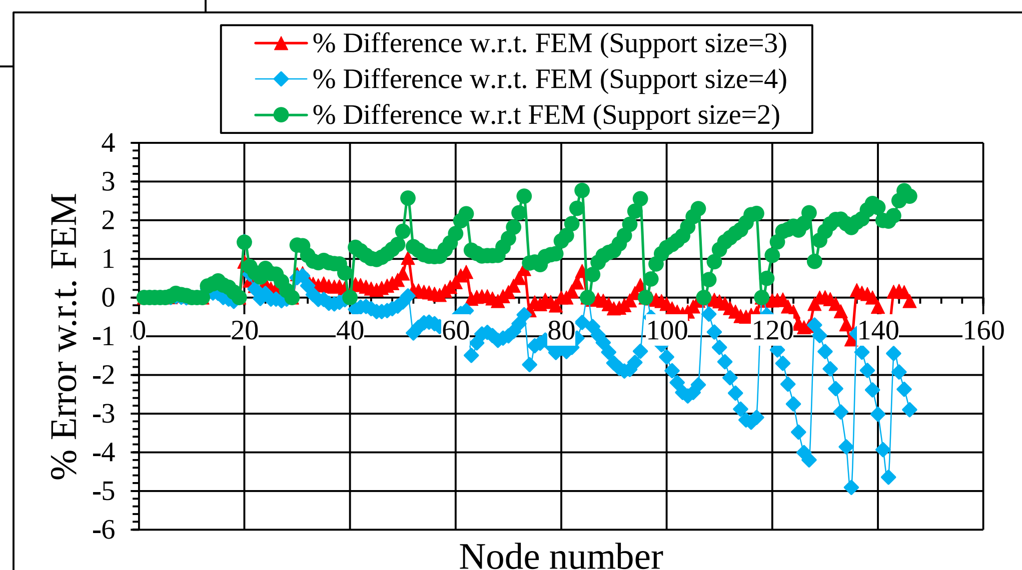
Monthly variation in head values at upsteam river boundary node (no .40)

Mfree model application (cont.)

Effect of time- step size



Effect of support for basis function



Conclusions

- Mfree groundwater model showed good agreement with analytical head values for both 2D synthetic problems.
- Since model performing well with support size between 2 to 3 hence it reduced the dependency on grid- based solution for its calibration.
- Developed model showed higher accuracy with increasing nodal density.

Thank You