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Digital filters for application to tidal groundwater time series

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Presentation contents

1. Purpose of study

2. Introduction to nonrecursive digital filtering

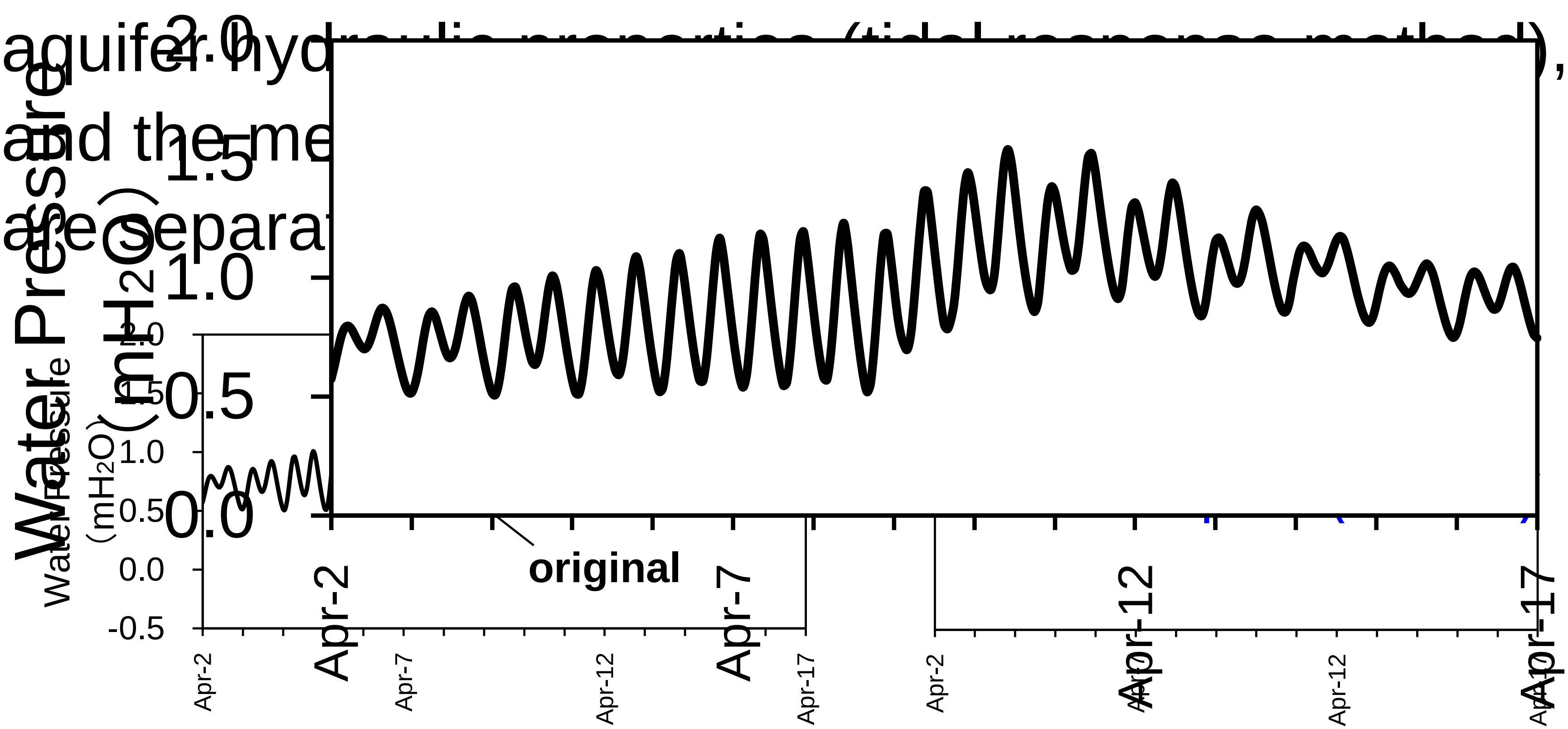
3. Comparative study of digital filters
   a. Running-mean filters
   b. Selected-mean filters
   c. Cosine filters using windows
   d. Optimized tide-killer filters

4. Conclusion

1. Purpose of Study

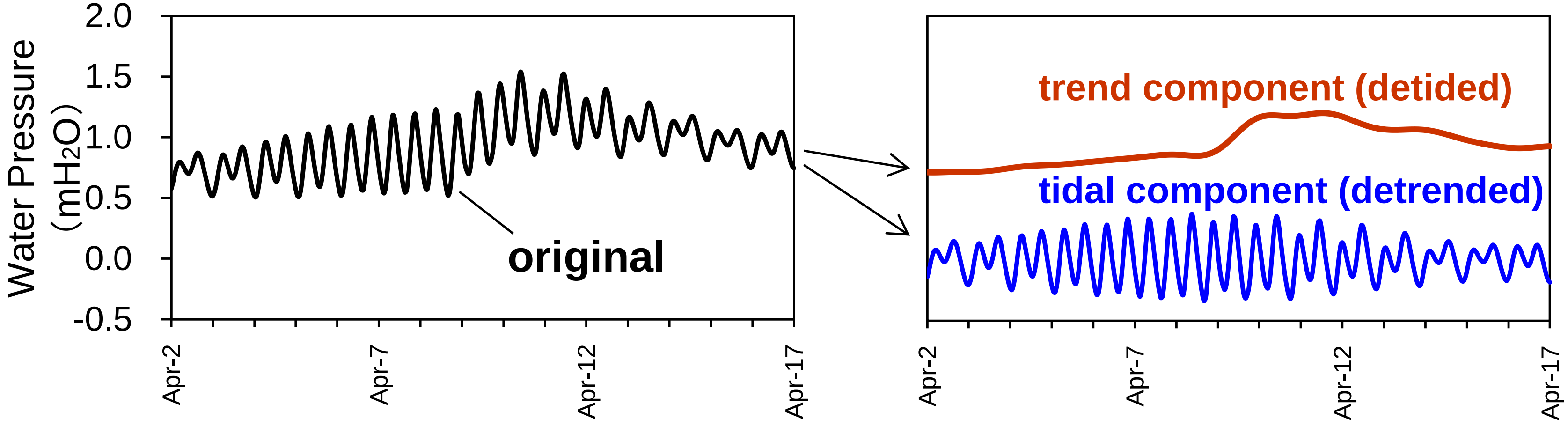
Groundwater observation time-series data collected from coastal areas often include significant **tidal oscillations** with **longer-period fluctuations**.

Tidal oscillations can be used to investigate aquifer hyd... (tidal response method), and the me... are separa...



1. Purpose of Study

Conversely, when investigating the **long-period fluctuations**, e.g., in water table to study the effect of recharge, tidal signals only make a nuisance.

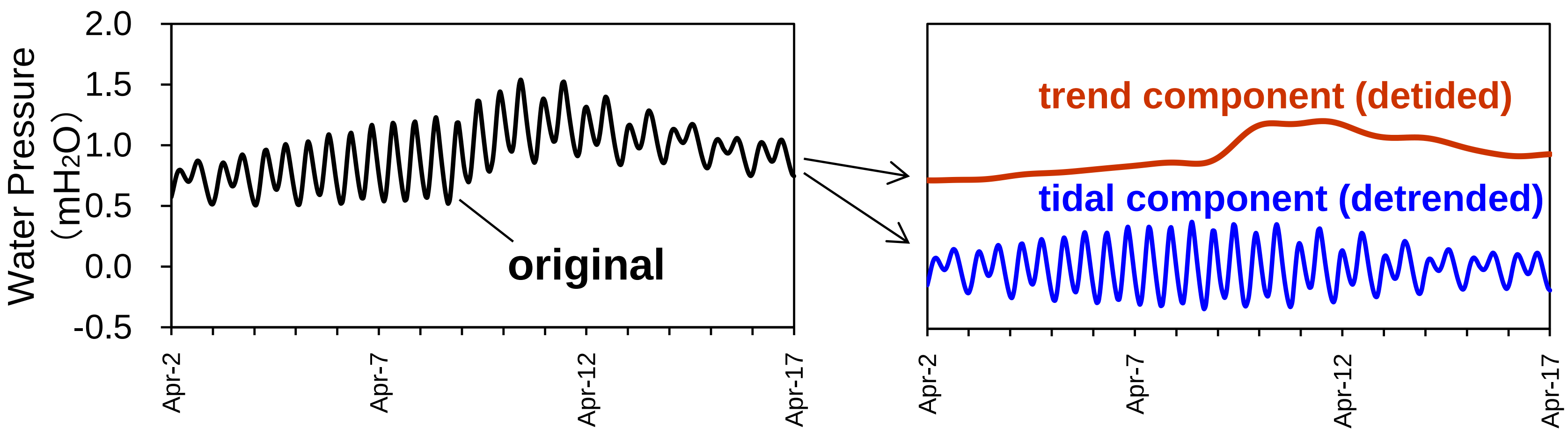


1. Purpose of Study

Conversely, when investigating the **long-period fluctuations**, e.g., in water table to study the effect of recharge, tidal signals only make a nuisance.

Comparative study was made on "**digital low-pass filters**" to find simple techniques for **eliminating (semidiurnal to diurnal) tidal components**.

*Corresponding **high-pass** filters extract tidal components.



2. Nonrecursive Digital Filtering

The studied digital filters are all nonrecursive type.

Nonrecursive digital filtering is represented by the linear formula:

$$y_n = \sum_{k=-m}^{m} W_k \cdot x_{n+k}$$

where $\boldsymbol{y_n}$ are the output data, $\boldsymbol{x_{n+k}}$ the input data, and $\boldsymbol{W_k}$ the weighted terms of the filter.

(e.g., 5-point running averaging is identical to application of a digital filter composed of: $W_k = \{0.2, 0.2, 0.2, 0.2, 0.2\}$.)

input

...	x_{n-4}	x_{n-3}	x_{n-2}	x_{n-1}	x_n	x_{n+1}	x_{n+2}	x_{n+3}	x_{n+4}	...

filter ←

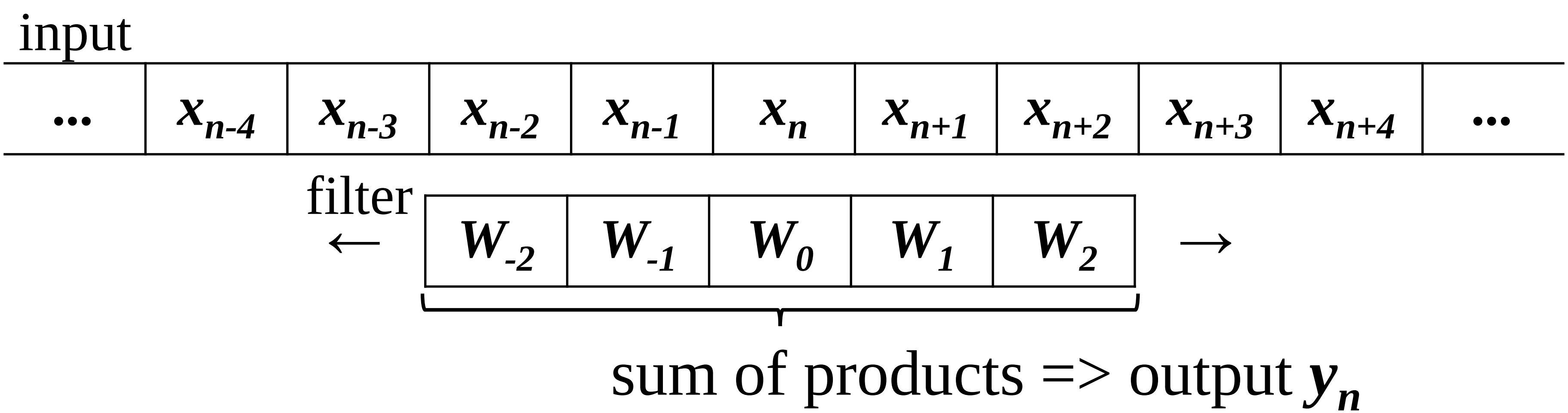
W_{-2}	W_{-1}	W_0	W_1	W_2

→

sum of products => output y_n

2. Nonrecursive Digital Filtering

Nonrecursive digital filtering of electromagnetic time-series data is easily achieved on a sheet of prevalent spreadsheet software using a built-in function (SUMPRODUCT) for calculating the sum of products of corresponding terms of two number sequences.

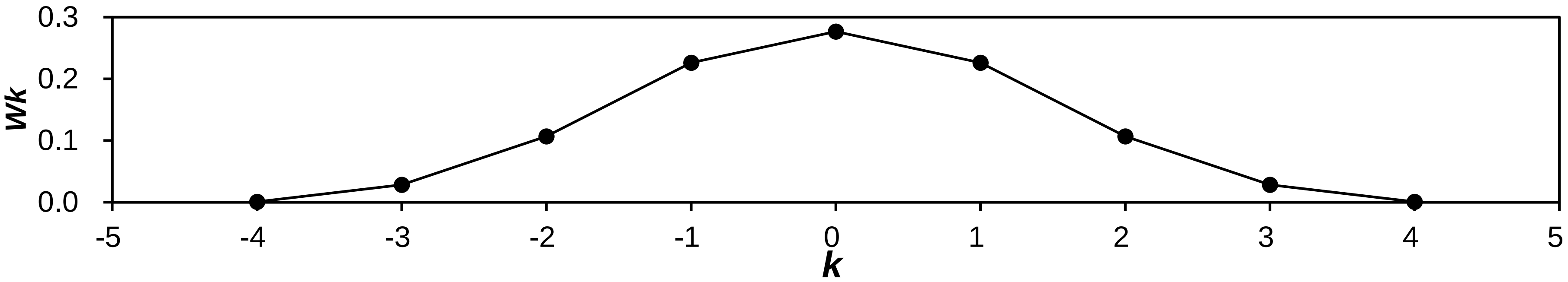


2. Nonrecursive Digital Filtering

The studied digital filters are all **symmetric filters** that use an odd number of input data points:

$$y_n = \sum_{k=-m}^{m} W_k \cdot x_{n+k} \qquad (W_{-k} = W_k)$$

Various filters can be made by specifying the filter length (number of terms) and the values of terms.



filter	W_4	W_3	W_2	W_1	W_0	W_1	W_2	W_3	W_4

2. Nonrecursive Digital Filtering

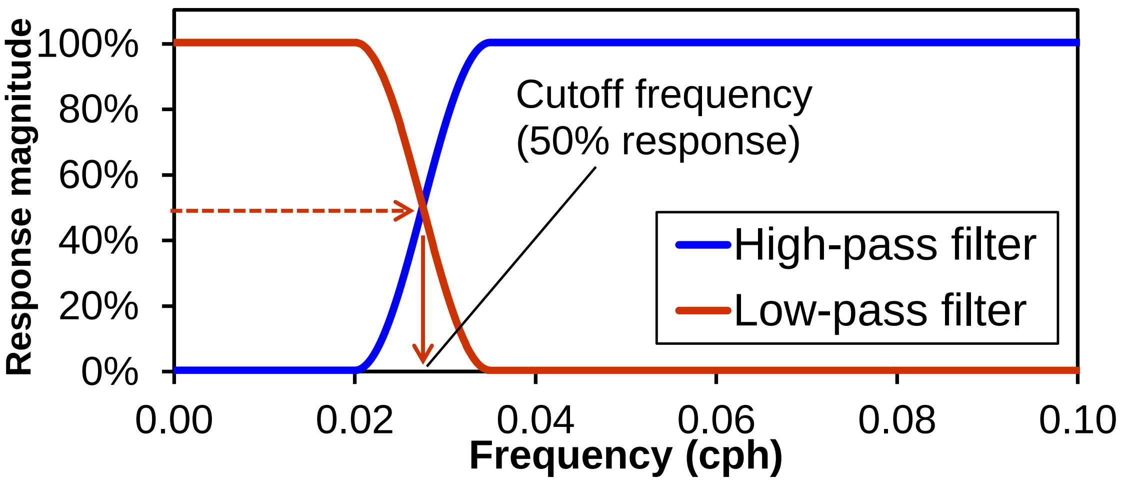
Performance of a symmetric digital filter is specified by the **filter response function**:

$$R(\omega) = W_0 + 2 \cdot \sum_{k=1}^{m} W_k \cos(\omega k)$$

that indicates the ratio of output amplitude to input amplitude of signals for frequency $\boldsymbol{\omega}$.

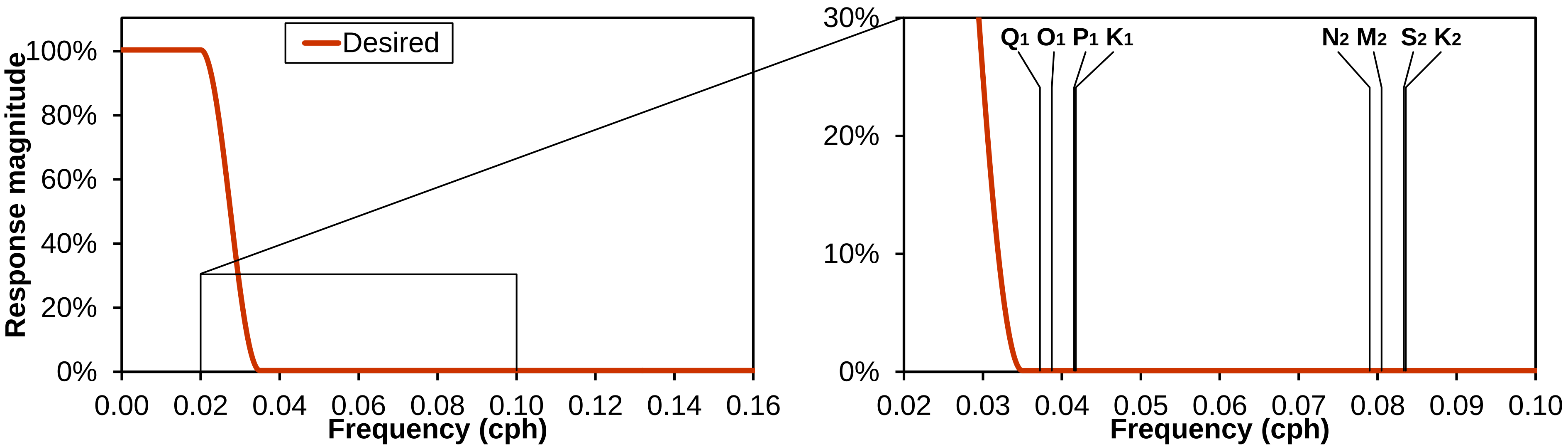
* time-series sampling interval is assumed to be 1 hour.

A **low-pass filter** shows responses near unity at low frequencies, and nearly zero at high frequencies.



2. Nonrecursive Digital Filtering

Desired filter response in the present study is:



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Desired filter response in the present study is:

(a) near unity for the frequencies under 0.02 cph
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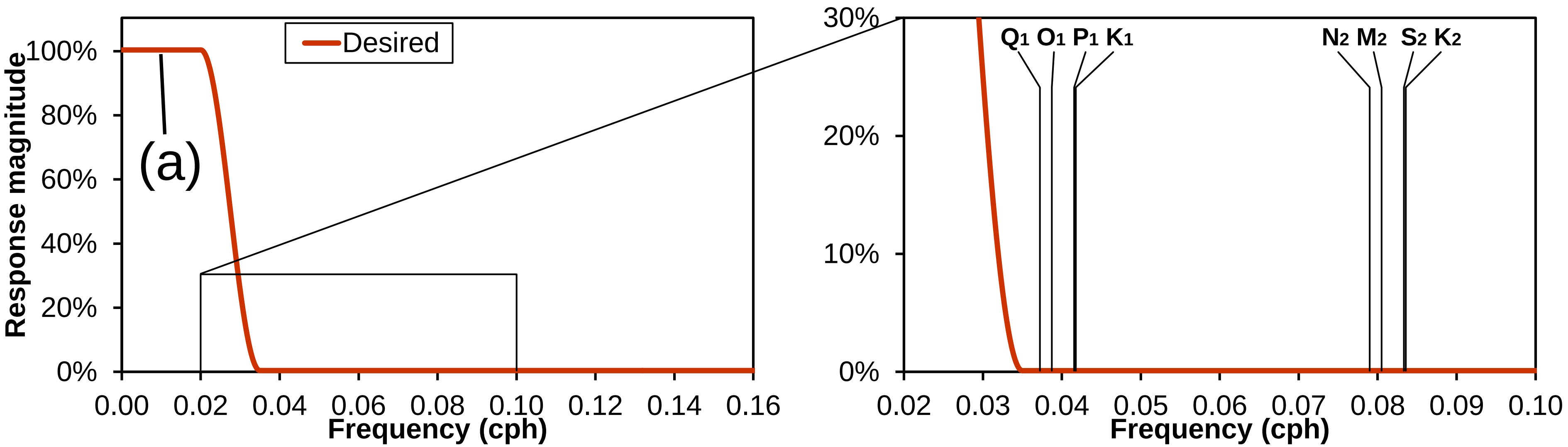

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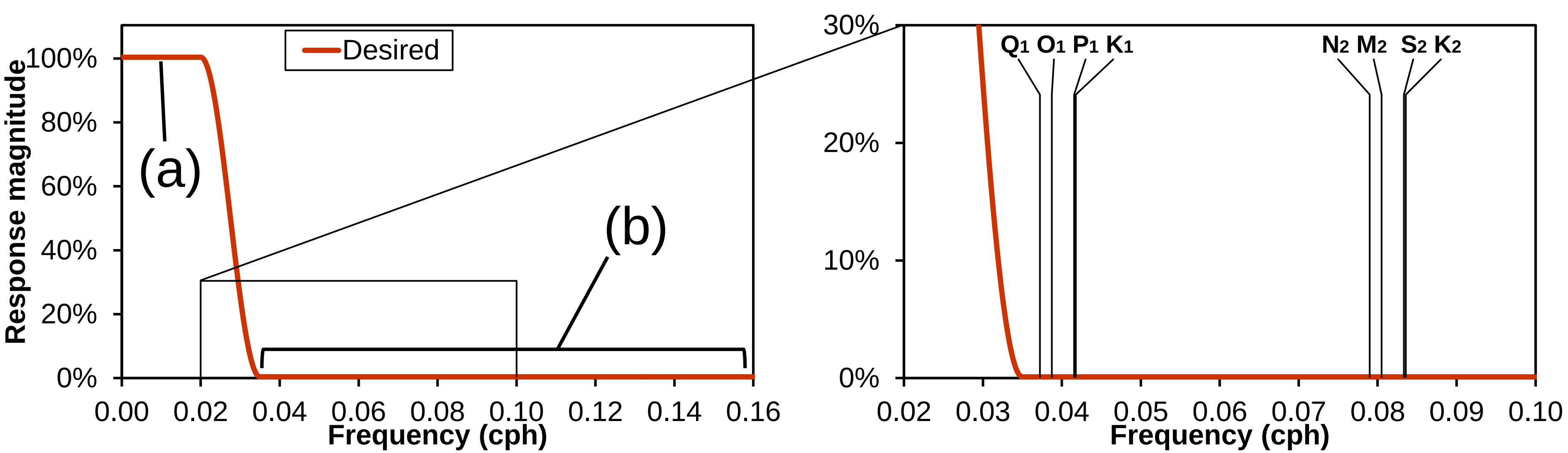

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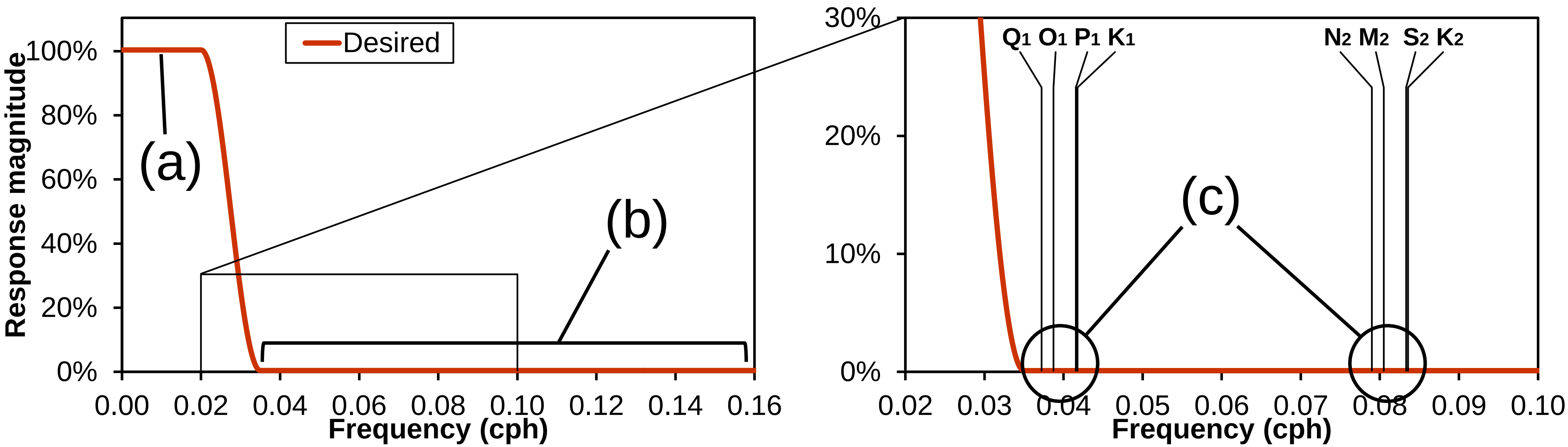
(a) near unity for the frequencies under 0.02 cph (periods longer than ca. 2 days),

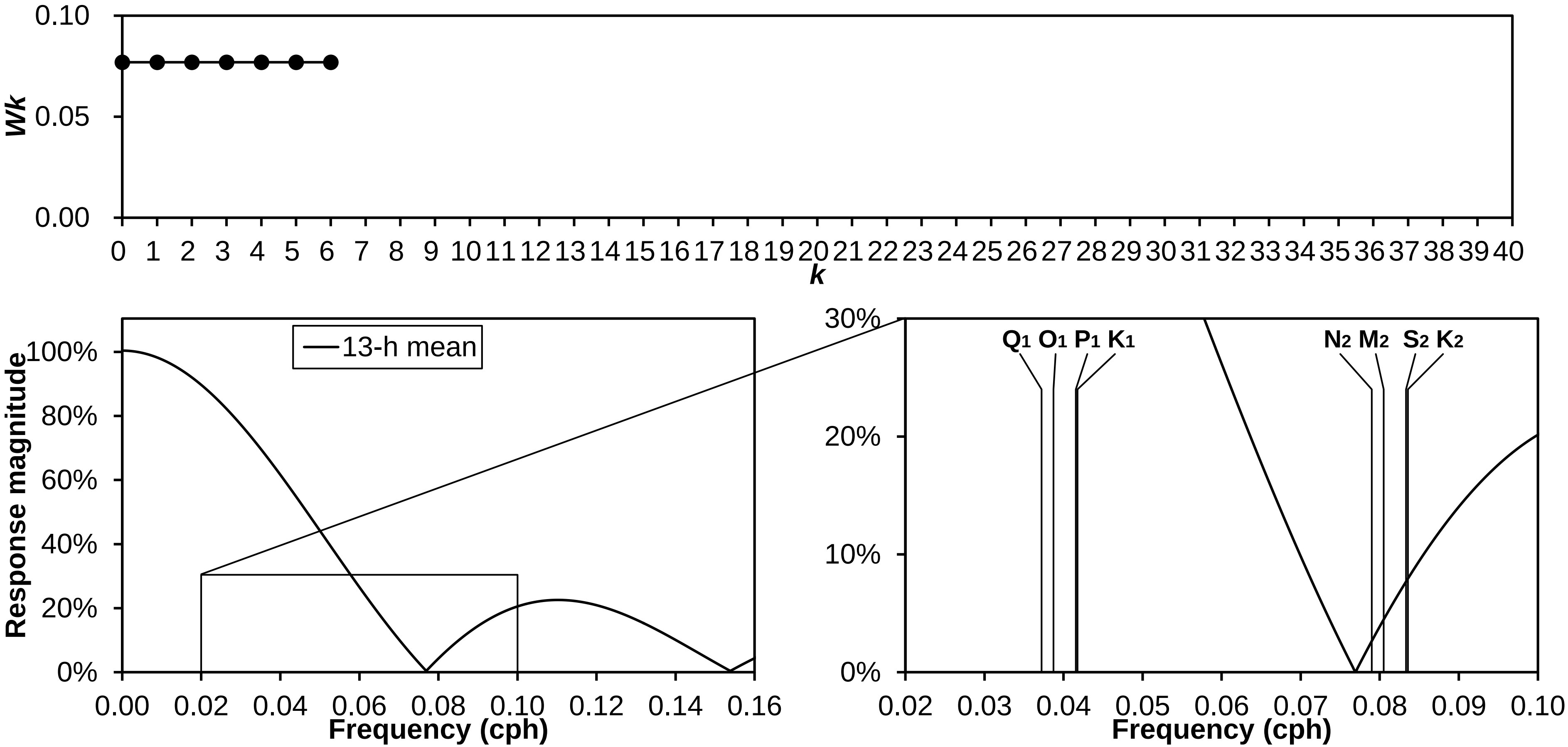
(b) near zero for the frequencies of diurnal tides and higher (> ca. 0.035 cph), and

(c) perfect zero for the eight major tidal constituents (Q_1, O_1, P_1, K_1, N_2, M_2, S_2, K_2).



3a. Running-mean filters

Running-mean filters are the simplest low-pass filters. The length can be variously set.



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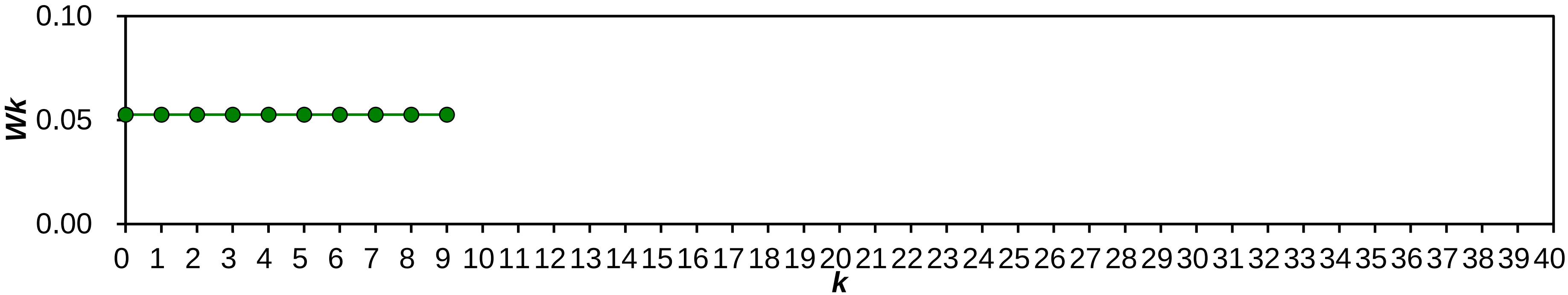

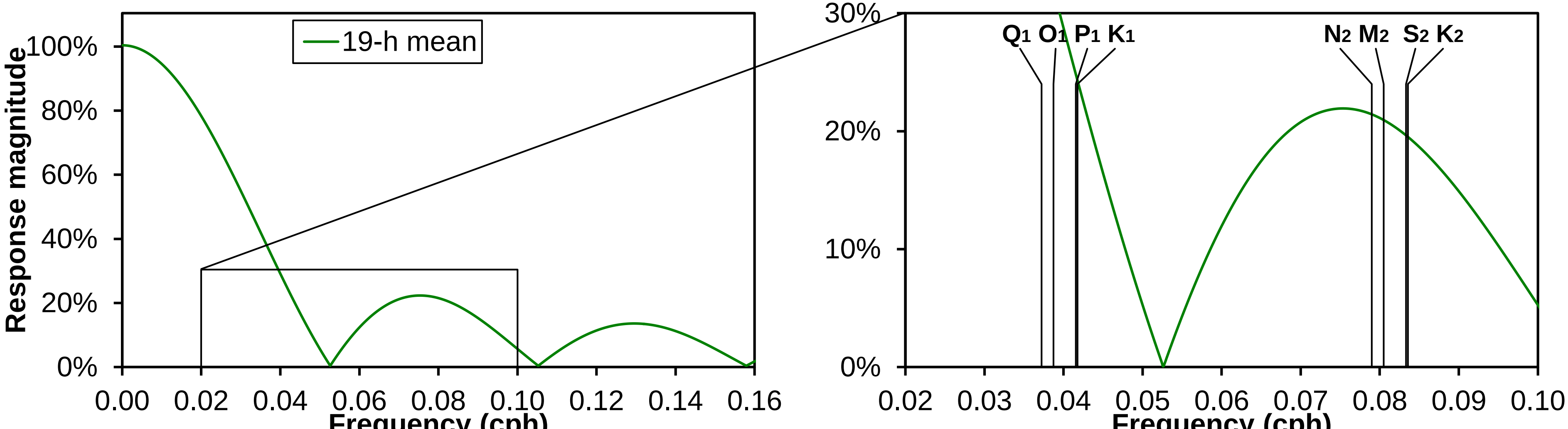

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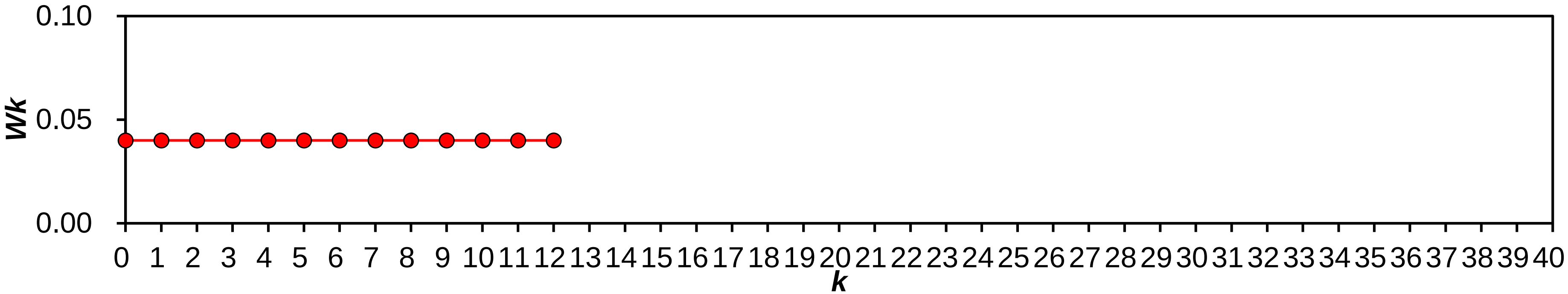

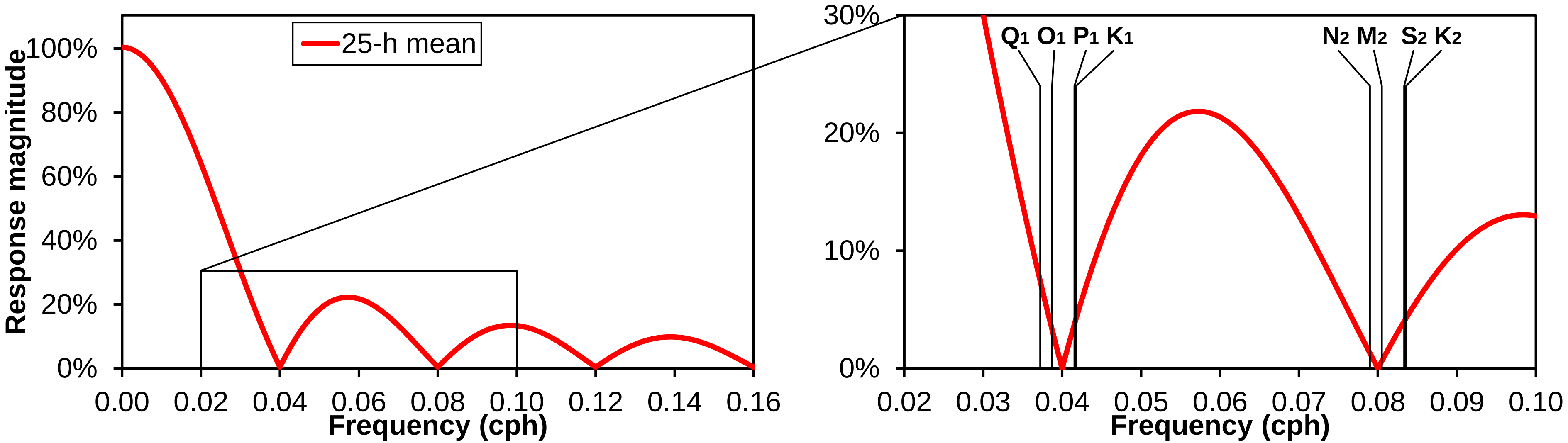

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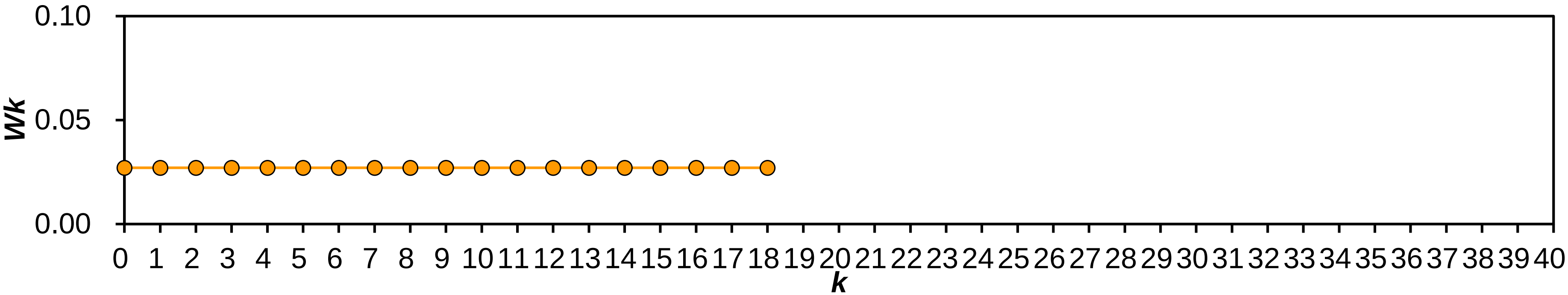

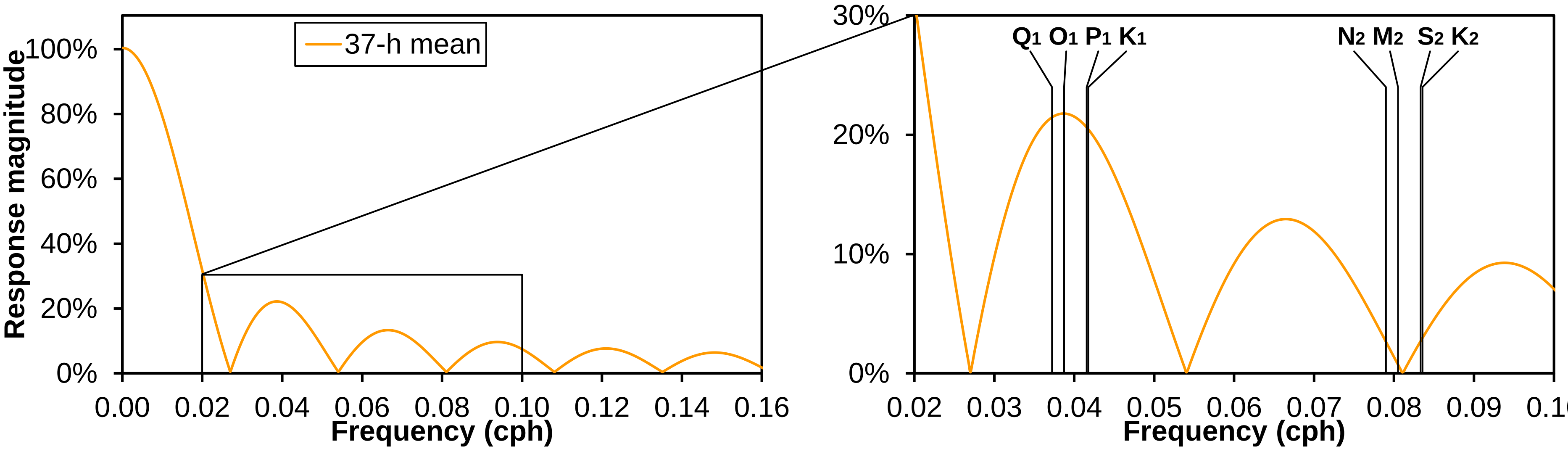

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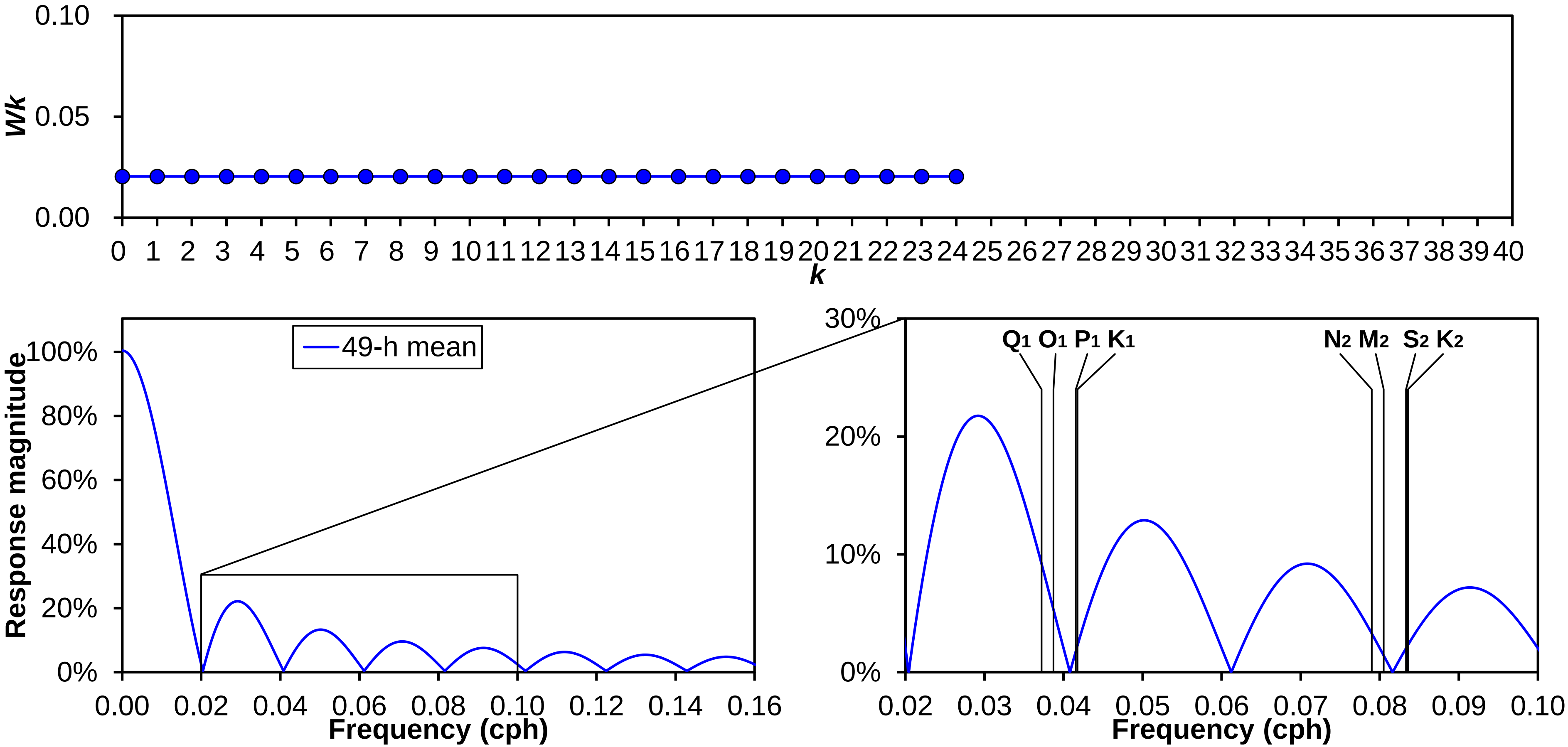
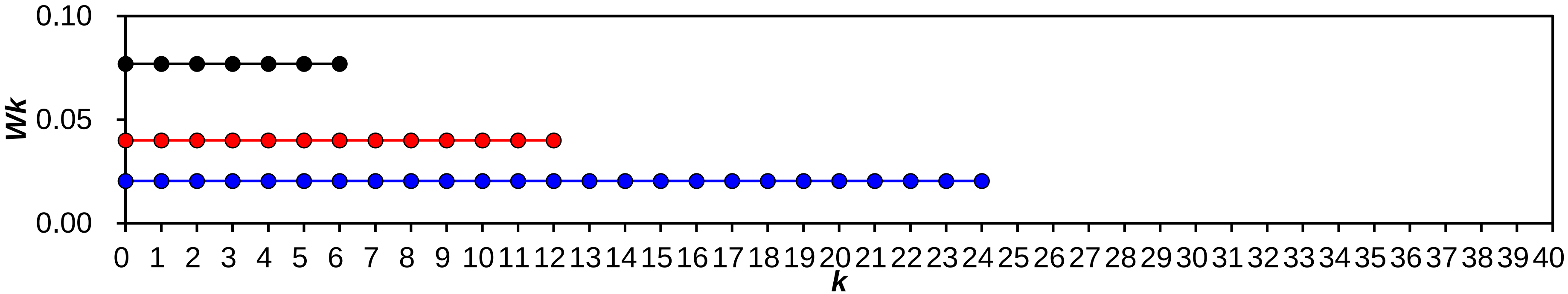
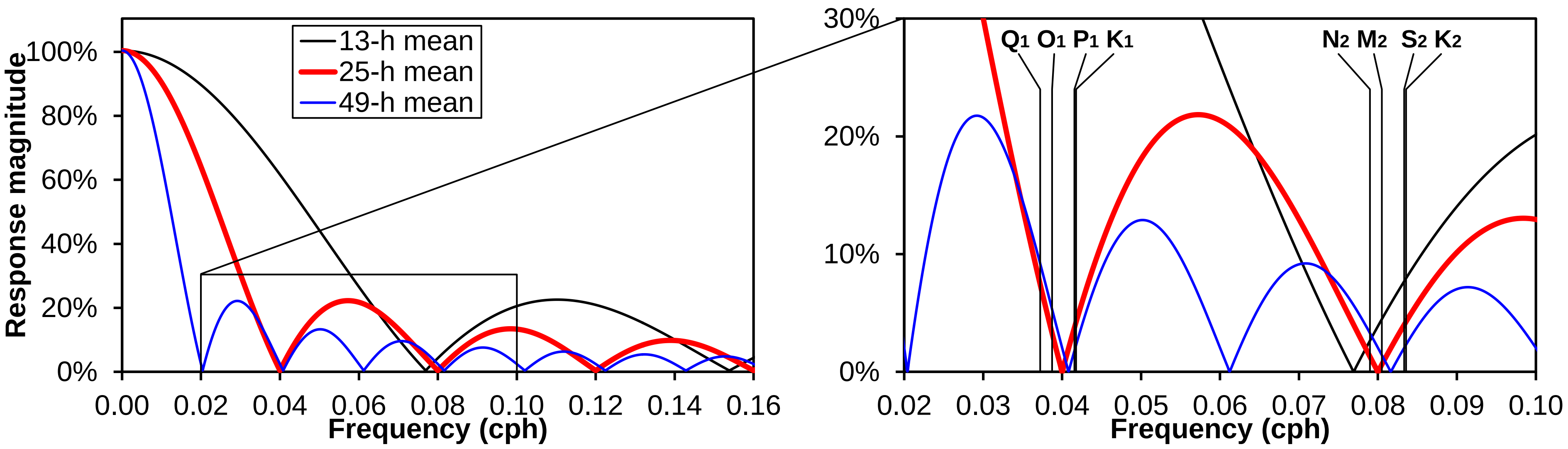


3a. Running-mean filters

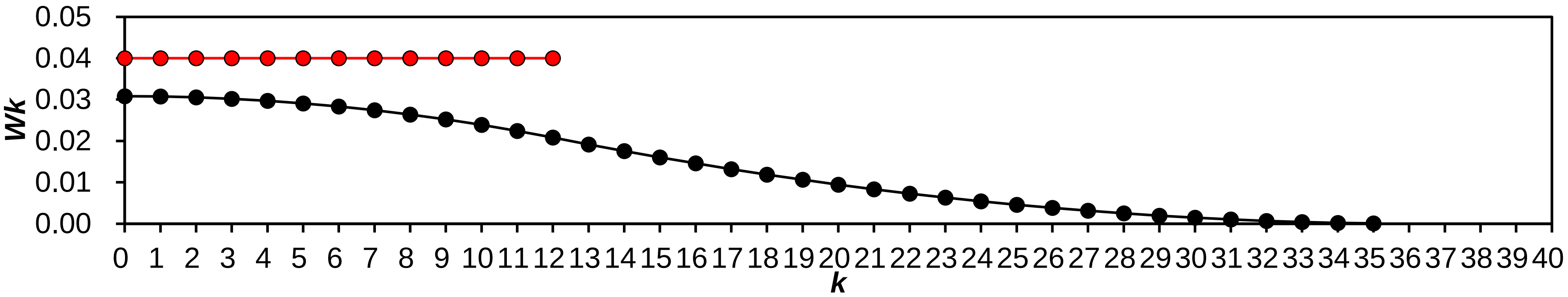
Running-mean filters are the simplest low-pass filters. The **25-hour running-mean filter** effectively suppresses diurnal and semidiurnal tides.

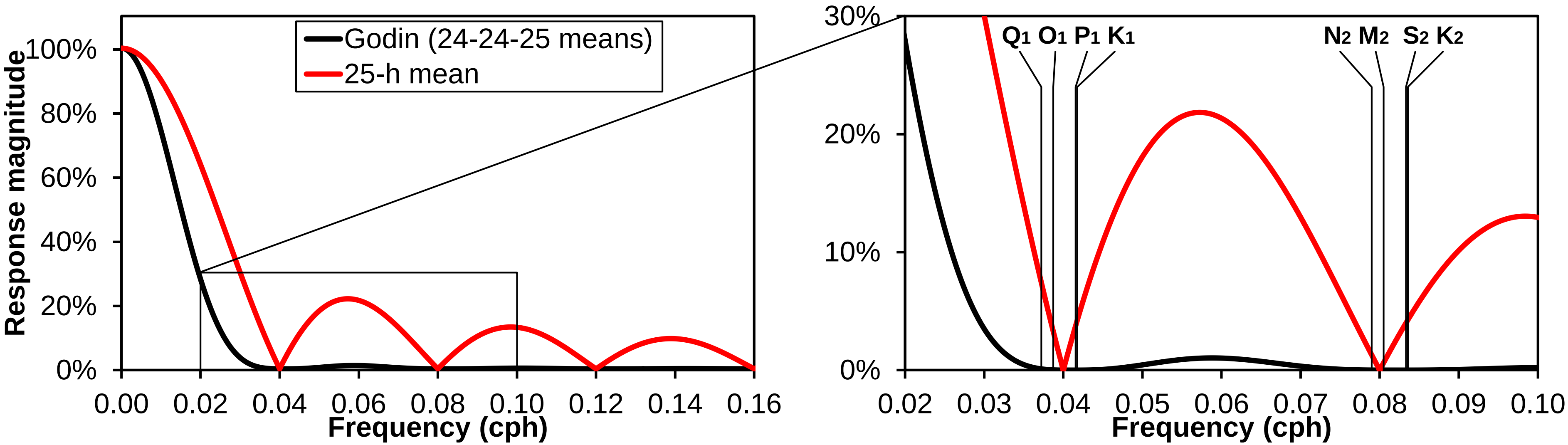




3a. Running-mean filters

Godin's (1966) **24-24-25 cascaded running-mean filter** has been used in oceanography to suppress diurnal tides and higher frequencies.



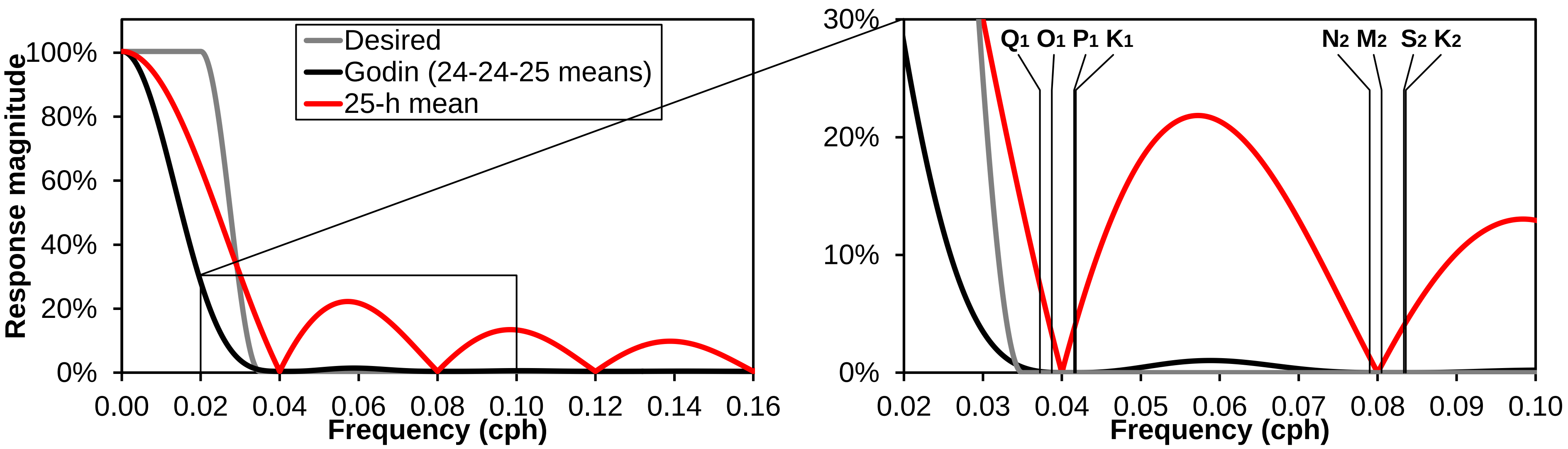


3a. Running-mean filters

Running-mean filters are the simplest low-pass filters that can be used for suppressing tides, but

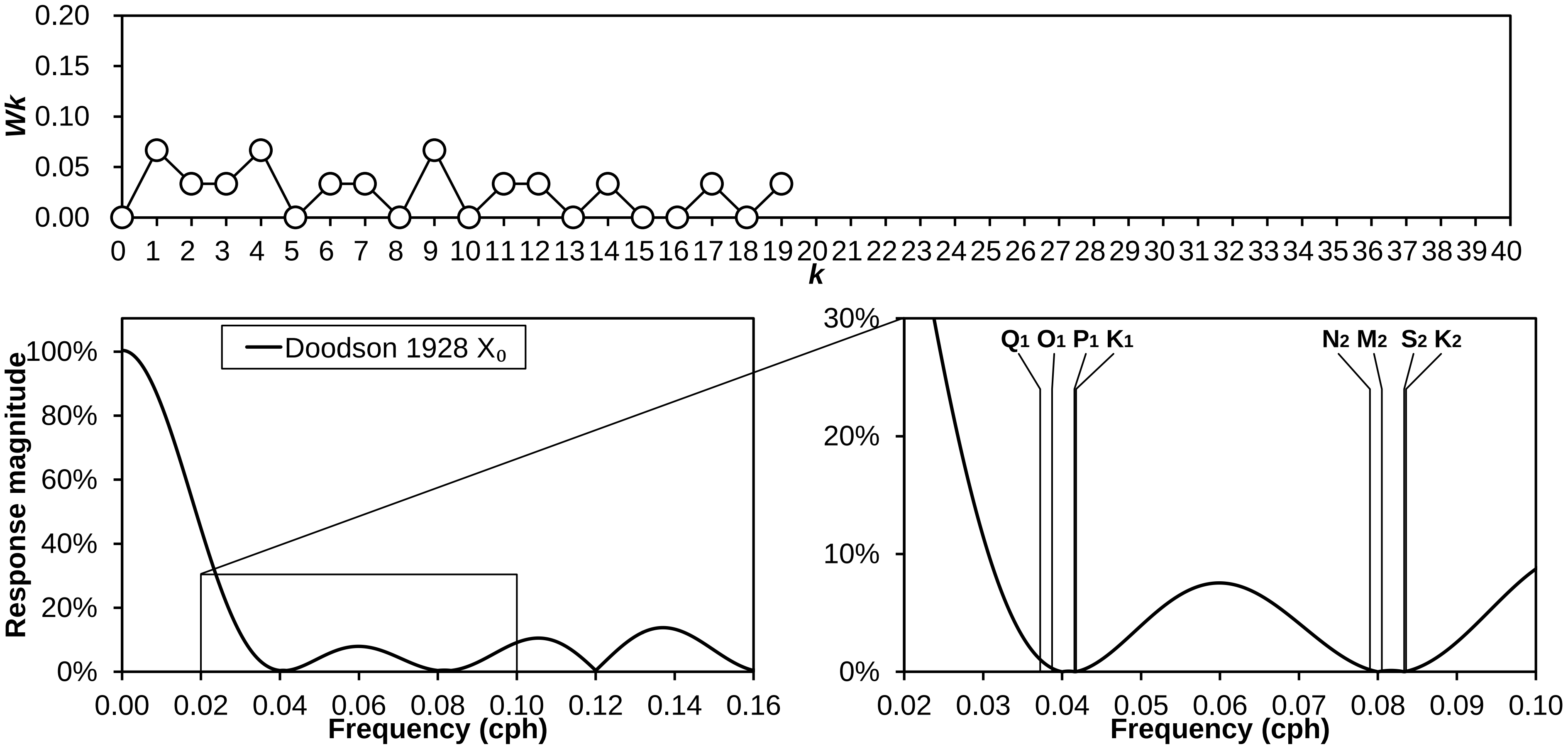
(a) responses for low frequencies (< 0.02 cph) are undesirably smaller than unity, and

(b) responses for high frequencies (>0.035 cph) largely deviate from zero for the simple (non-cascaded) running-mean filters.



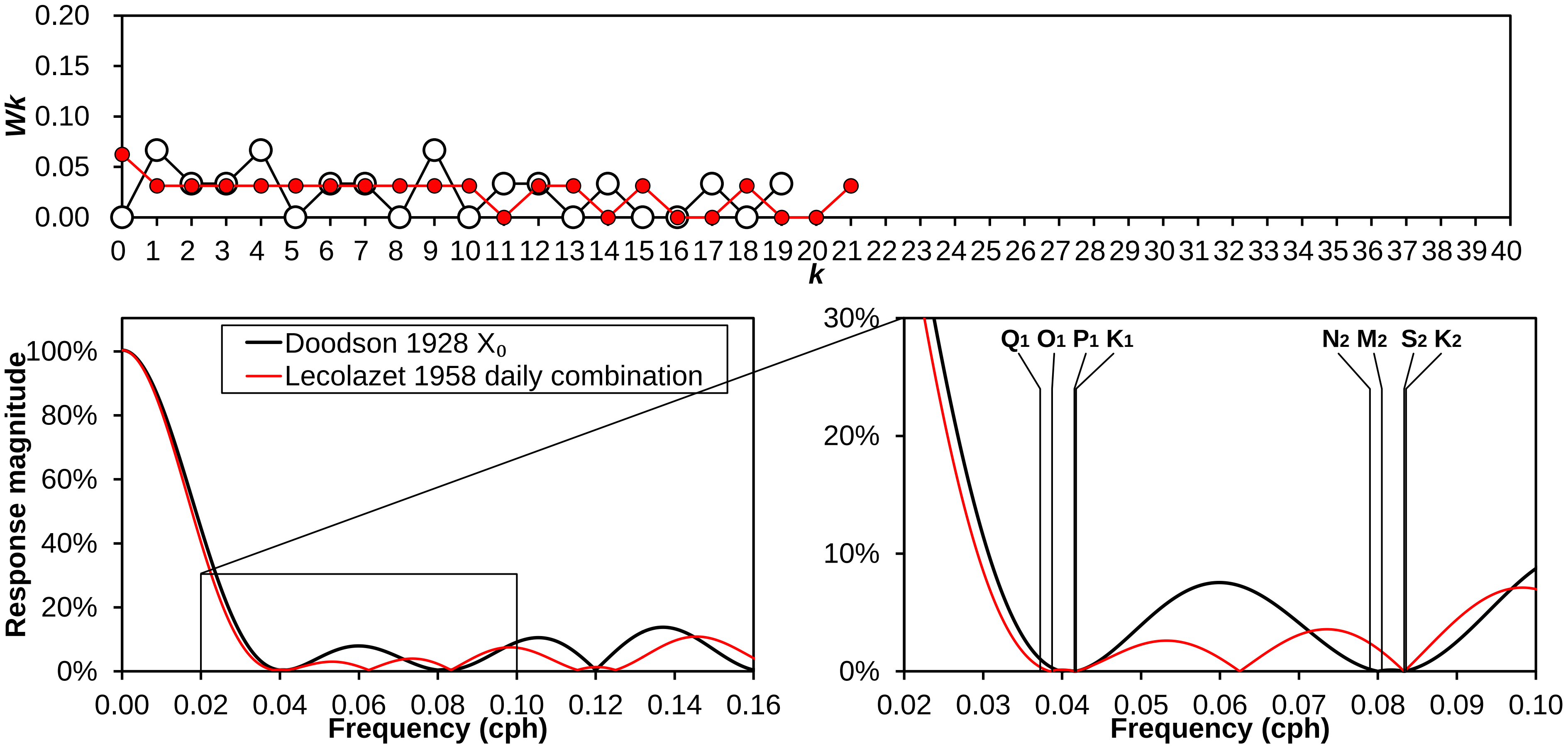
3b. Selected-mean filters

Doodson's (1928) **X_0 filter** is one of the earliest and most widely used filters in oceanography to suppress diurnal and semidiurnal tides.



3b. Selected-mean filters

A few dozen of other selected-mean filters have been presented in oceanographic and geodetic literature.



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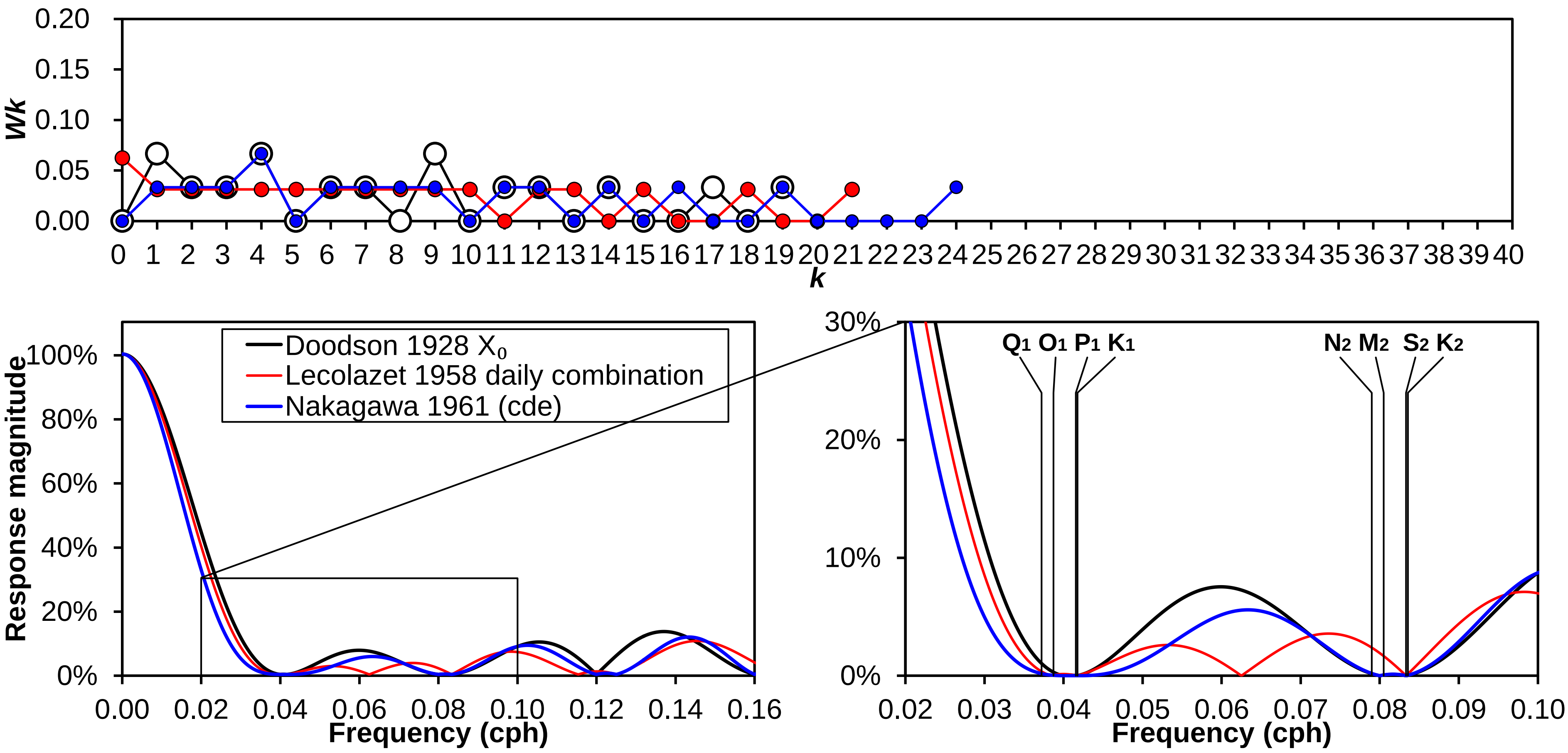

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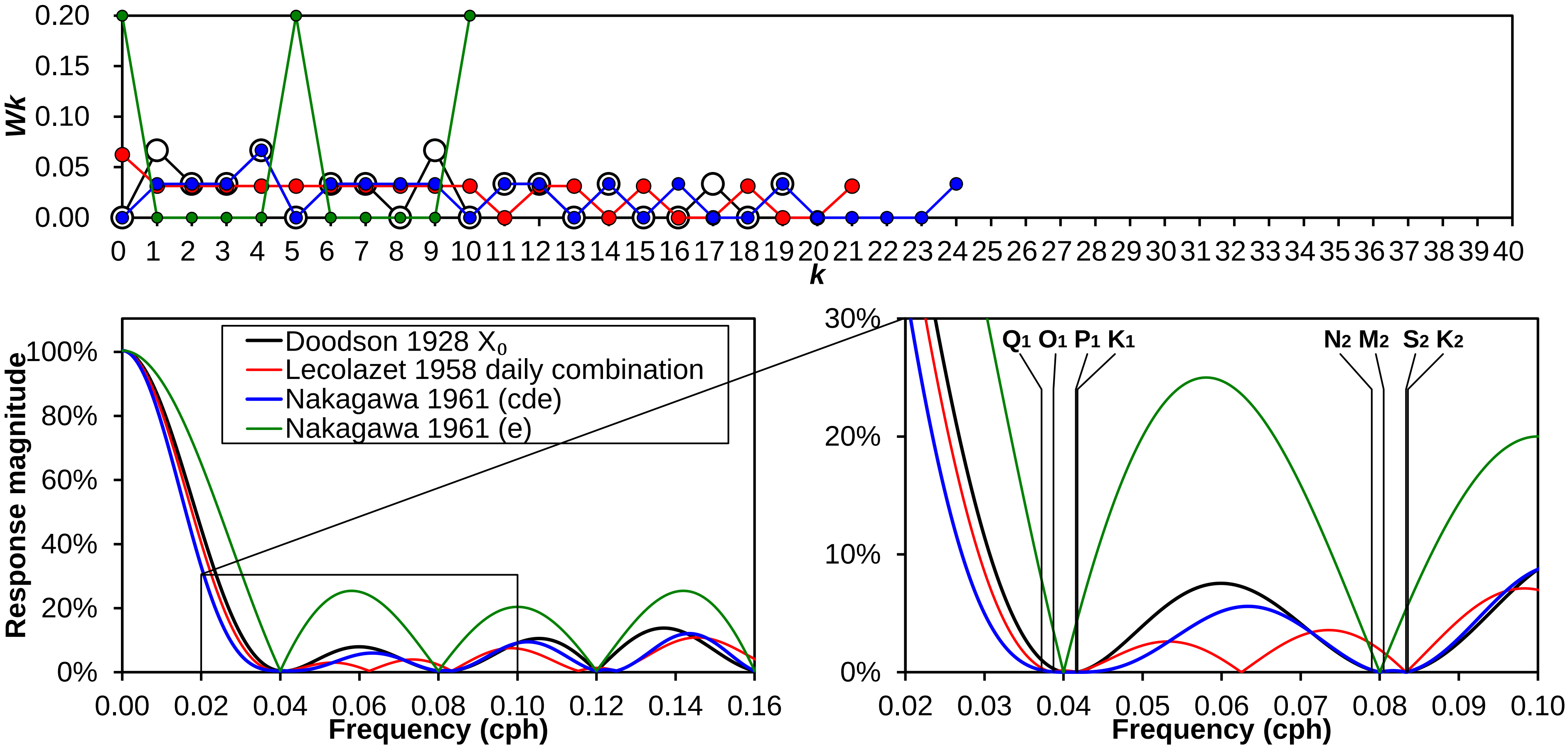
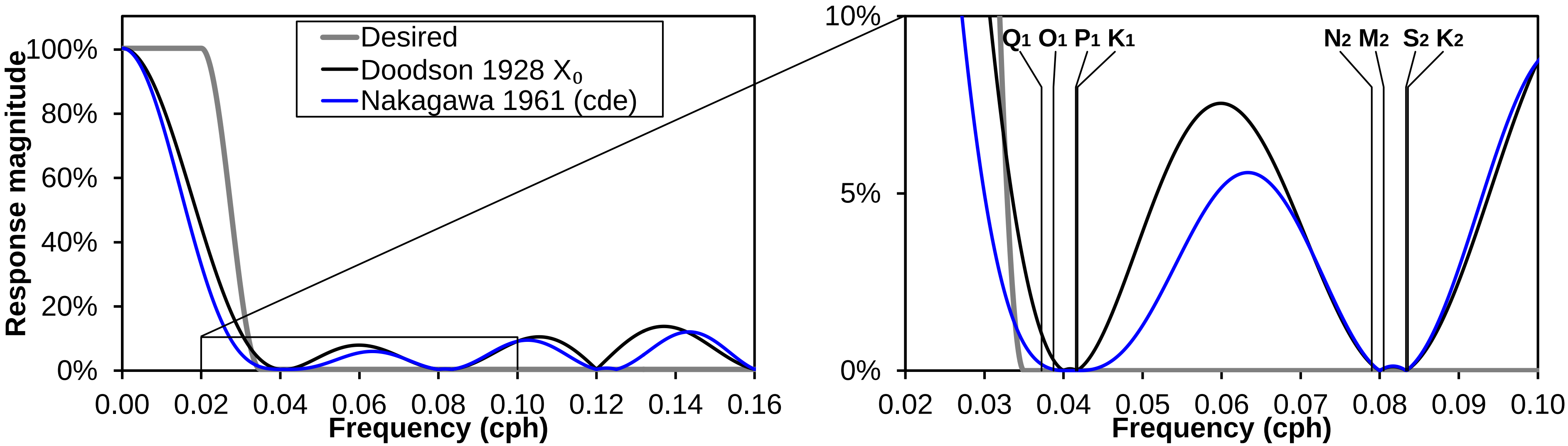


3b. Selected-mean filters

Selected-mean filters have been developed for the limited computational resources, and

(a) responses for low frequencies (< 0.02 cph) are undesirably smaller than unity, and

(b) responses for high frequencies (>0.035 cph) are not perfect for eliminating major tides.

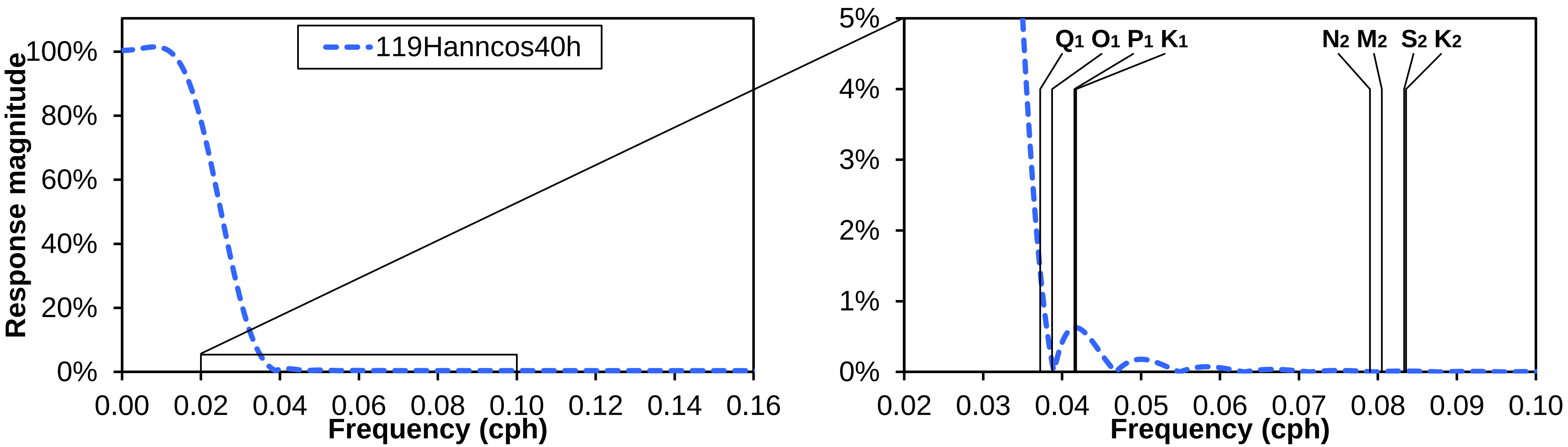


3c. Cosine filters using windows

In oceanography **cosine filters using windows** have been proposed (Emery & Thomson, 2001).

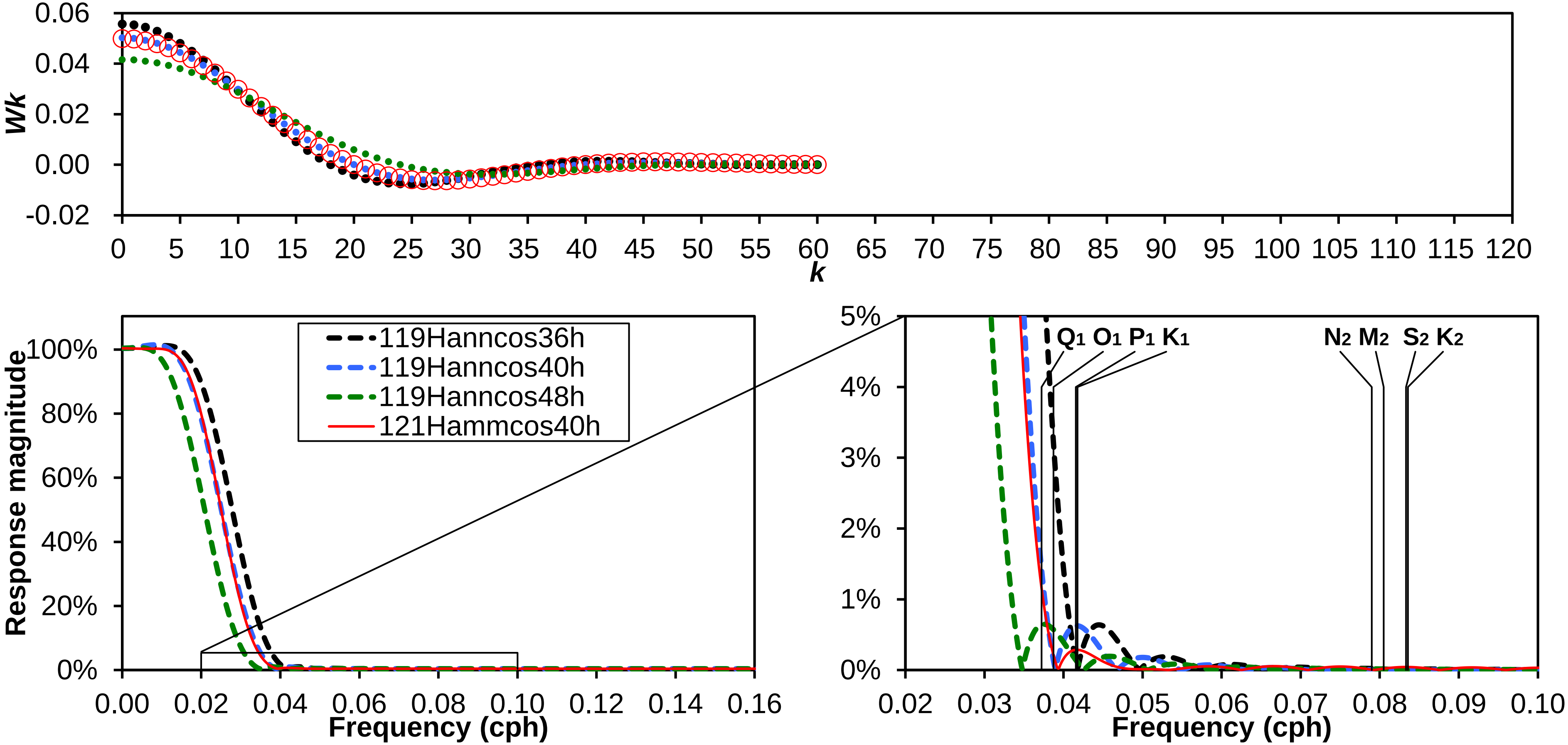
An example is the 119-hour-long filter using "von Hann window" with cutoff period of 40 hours:

$$W_k = \frac{0.5 \cdot \left[1 + \cos(k\pi / 60)\right] \cdot \sin(k\pi \cdot 2/40)}{(k\pi \cdot 2/40)} \qquad (0 \le k < 60)$$



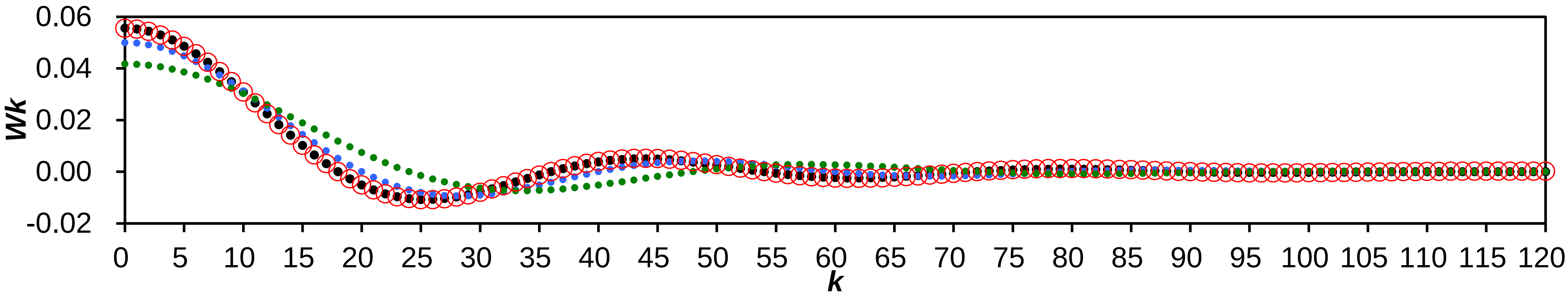
3c. Cosine filters using windows

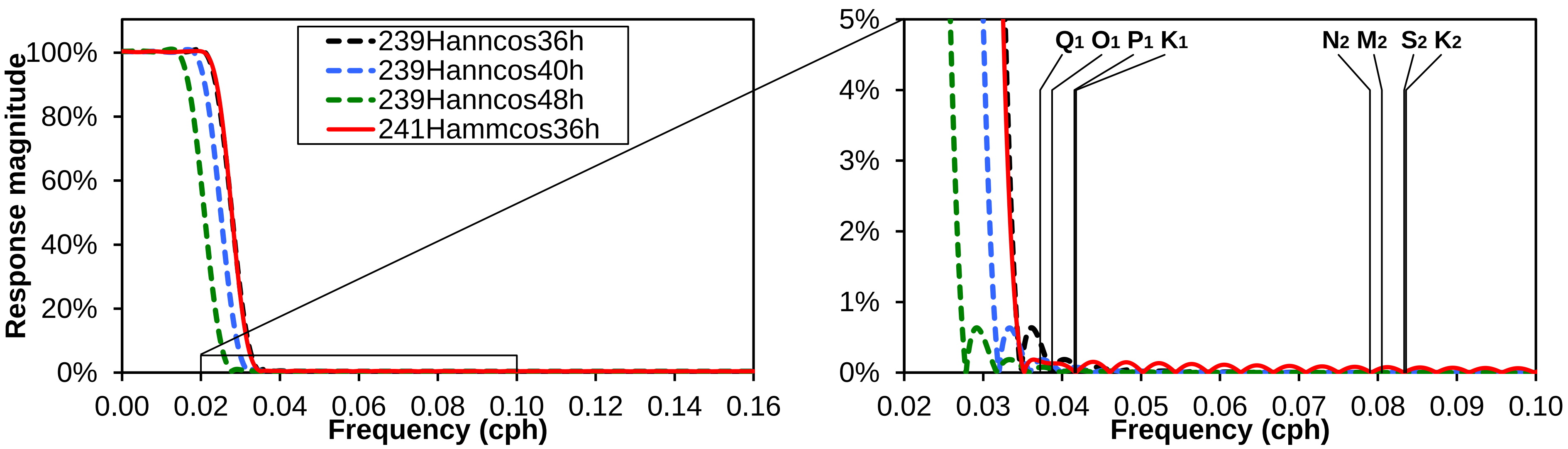
Setting the cutoff period as different values or using different windows (e.g., “Hamming window”) makes other cosine filters.



3c. Cosine filters using windows

Longer cosine filters using windows can be desirable low-pass filters, e.g., **241-hour-long filter using Hamming window with cutoff period of 36 h**.

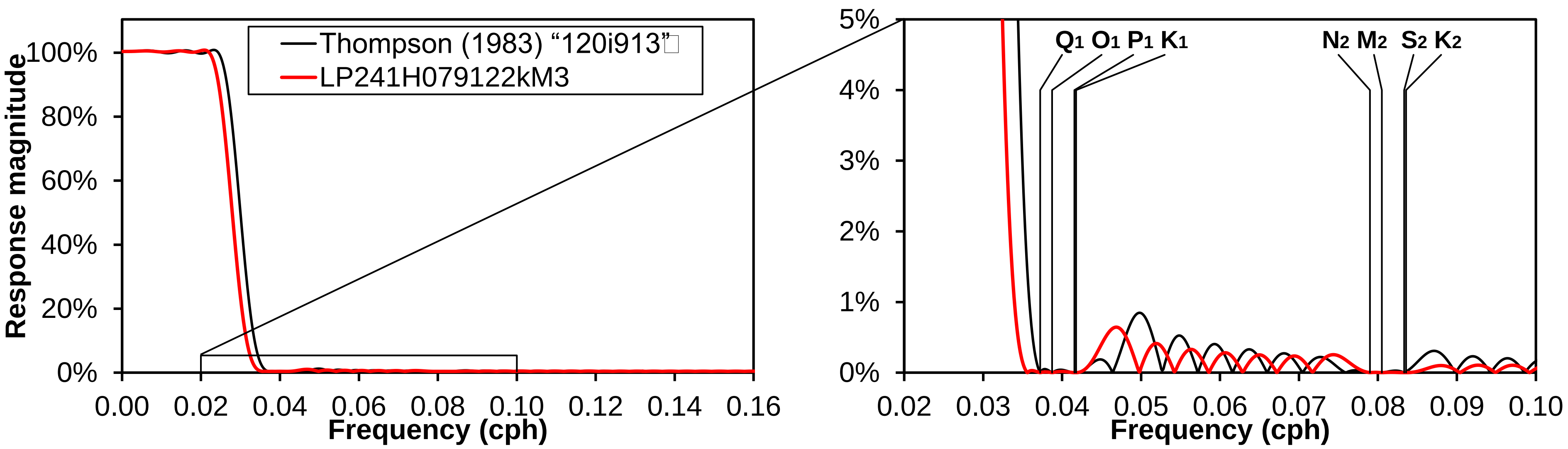




3d. Optimized tide-killer filters

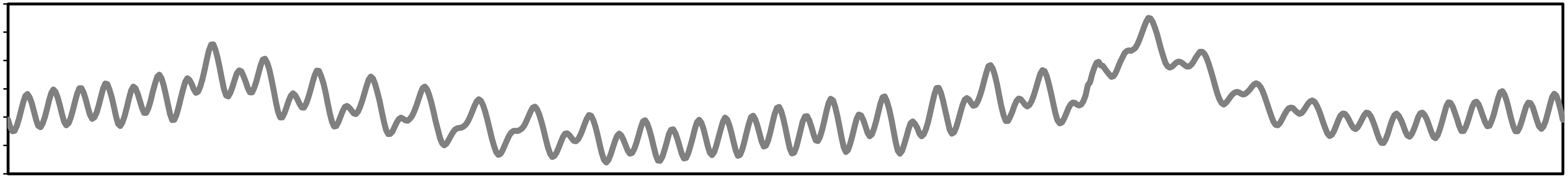
Thompson (1983) proposed a method to make digital low-pass filters imposing zero responses at arbitrary frequencies.

The method produced **optimal "tide-killer" low-pass filters**, with zero responses for the eight major tides (Q_1, O_1, P_1, K_1, N_2, M_2, S_2, K_2).



4. Conclusion

Optimized **tide-killer low-pass filters** can eliminate diurnal and shorter-period tidal signals from time-series data.



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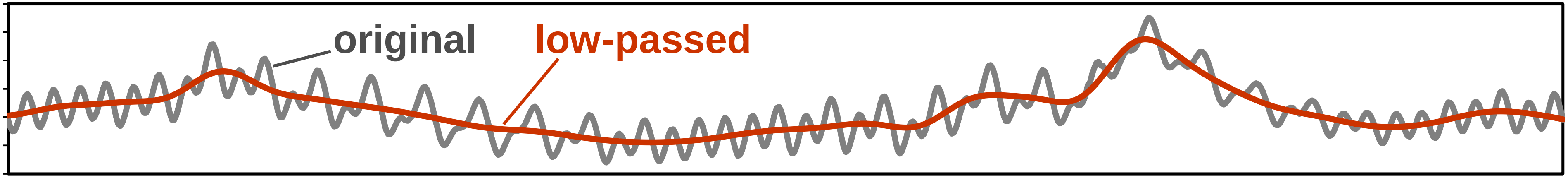
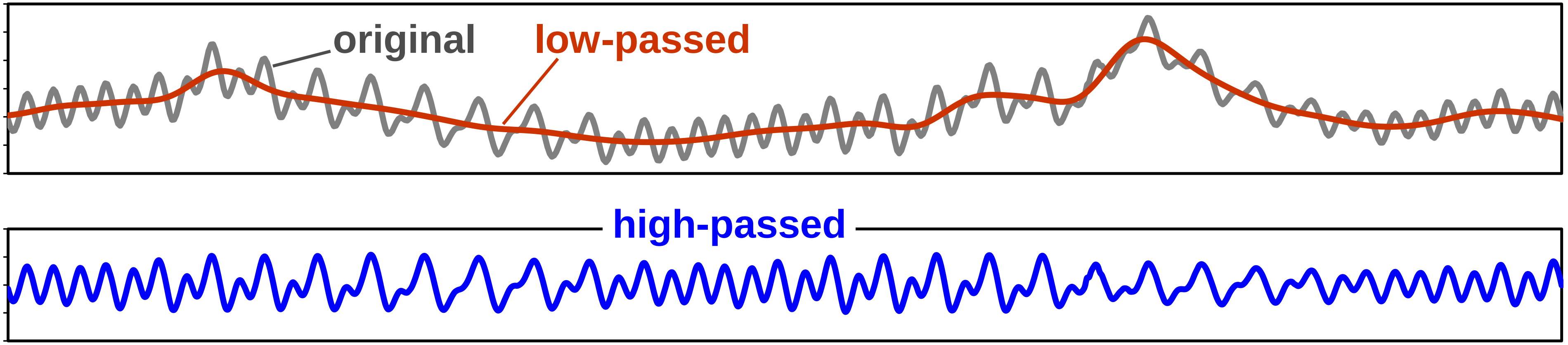
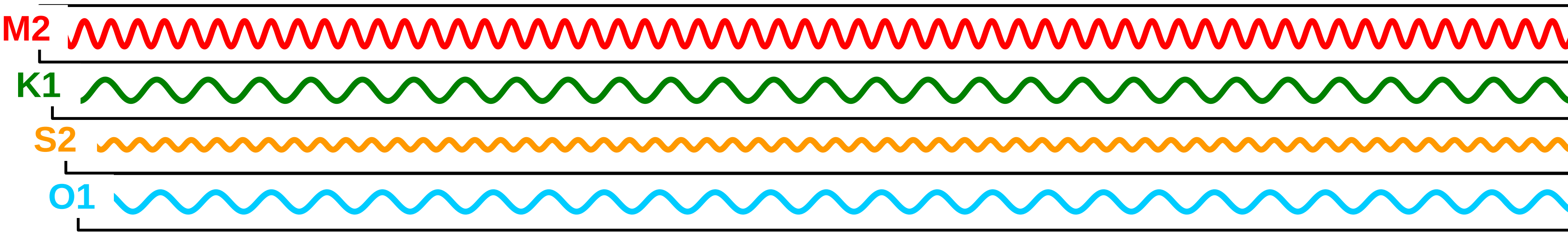


4. Conclusion

Corresponding **high-pass filters** can accurately separate semidiurnal to diurnal major tides, and the high-passed separated tidal time series can be used to accurate isolations of major tidal constituents.





4. Conclusion

The nonrecursive **digital filtering** for separating tides and longer-period fluctuations (and also isolation of major tidal constituents) can be easily achieved by using prevalent spreadsheet software.

For reference:

<Separation of low- and high-frequency bands>

Digital filters to eliminate or separate tidal components in groundwater observation time-series data, Japan Agricultural Research Quarterly: JARQ, 50, 241–252.

<Isolation of components of specified frequency>

Improvements in a simple harmonic analysis of groundwater time series based on error analysis on simulated data of specified lengths, Paddy and Water Environment, doi: 10.1007/s10333-016-0525-3.